Physics 180D

Experiment #2

OBJECTIVE:

To study the dispersive nature of waves in shallow water, and to measure the damping of the waves. At high drive amplitudes a unique nonpropagating soliton can be generated and characterized.

Water Waves

Surface waves on water are characterized by a dispersive spectrum, where the phase velocity of the wave depends on the frequency. This means that a water-wave pulse will change shape as it propagates, since the different frequency components making up the pulse will travel with different speeds. We can measure this dispersion by looking at the resonant frequencies of surface wave modes in a long thin trough partially filled with water. At low frequencies the dispersion relation for shallow water waves is given by

$$\omega^2 = gk \tanh(kh) \tag{1}$$

where g is the gravitational acceleration and h is the depth of the water. For very long wavelengths such that kh << 1 the tanh function can be expanded, tanh(kh) ≈ kh, and in this limit the phase velocity becomes

$$c_{ph} = \omega / k = \sqrt{gh} \ , \tag{2}$$

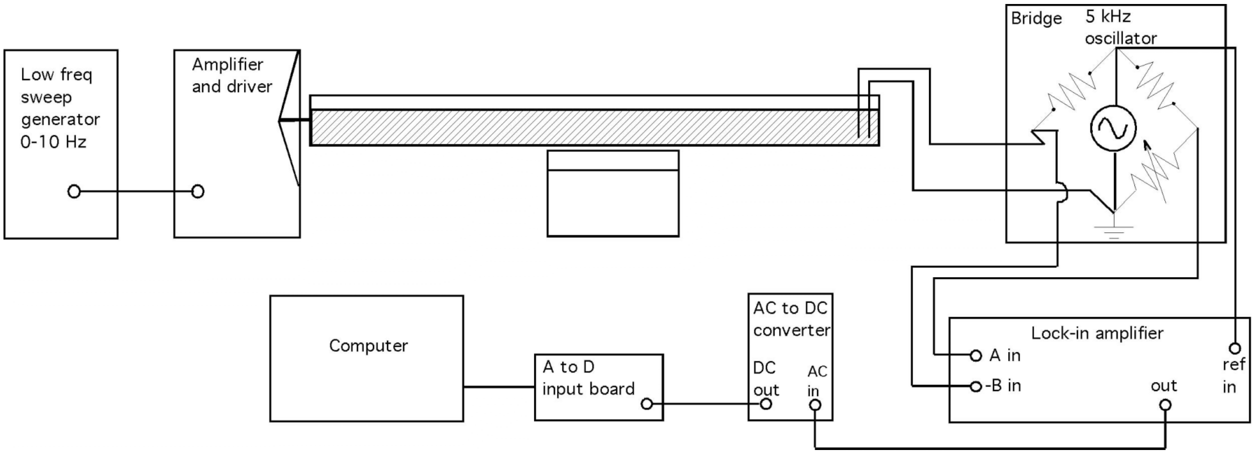
which is nondispersive. With increasing frequency however, the full dispersive result of Eq. (1) must be used in calculating the phase velocity. At higher frequencies where the wavelength becomes comparable to the capillary length of water (several millimeters), it is necessary to also include surface tension restoring forces in the dispersion relation,

$$\omega^2 = (gk + \frac{\sigma}{\rho} k^3) \tanh(kh) \tag{3}$$

where σ is the surface tension and ρ the liquid density. A technique for measuring σ is discussed in an appendix.

Apparatus

A schematic of the apparatus is shown below. An oscillator drives a low-frequency speaker that is attached to one end of the long glass channel filled with water to a depth h that can be measured with a ruler (try to level the channel so that the depth is uniform). The channel sits on a ball-bearing stage that allows it to move back and forth with the speaker. This generates a surface wave at the same frequency as the oscillator, and if the frequency matches a resonant mode a maximum in the oscillation of the surface will be generated. This is detected at the far end of the channel by using the fact that the resistance between two wires inserted in the water (which is made partially conducting by adding NaCl) will oscillate as the height of the water oscillates. A Wheatstone bridge circuit operating at 5 kHz detects the off-balance oscillating voltage corresponding to the motion of the surface at the few Hz frequency of the water wave. A lock-in amplifier is used to measure this voltage, and after being rectified with an AC to DC converter the DC rms amplitude (proportional to the wave amplitude) is then recorded as a function of the drive frequency using the computer digitizer and LabView. Since there is no connection between the sweep generator and the computer, they need to be synchronized manually by pushing the sweep start button at the same time the LabView VI is started.



Procedure

A. Fill the channel to a depth between 2.5 and 3 cm with the sample water, which contains both salt and PhotoFlo (this allows the water to wet the glass walls and avoids the additional restoring force on the wave discussed in the article by Heckerman *et al.* at the end of this write-up). Align and level the channel with the speaker so that the motion is smooth. Manually tune the frequency to the first few resonant modes, verifying they are at the expected values. Note that since the boundary conditions are that the fluid velocity at the ends must equal the velocity of the channel, only odd-integer values of $k = n\pi/L$ are generated.

B. With the amplitude adjusted so that the lock-in does not overload, sweep the frequency from 0.5-3.5 Hz in the maximum sweep time available, 1999 s. Repeat for the frequencies 3.5-7 Hz and 7-10 Hz (it will be necessary to boost the gain and possibly the drive amplitude at the highest frequencies, since the amplitudes of the modes drop rapidly with frequency).

C. Measure the damping coefficient α for the first 5 or 6 modes by manually tuning to each resonance, and then stop the channel motion by turning the speaker amplifier off. Record the time decay of the wave amplitude, which should vary as $A(t) = A_o \exp(-\alpha t)$.

D. Fun with solitons. Lay the speaker flat and screw in the aluminum plate to support the channel, so that it can oscillate in the vertical direction. Tune to a frequency just below twice that of the first mode in the direction across the channel, which has a wavenumber $k = \pi/W$ where W is the width of the channel (this should be a frequency of order 10 Hz). Increase the drive amplitude while periodically oscillating the water at the center of the channel towards the sides with a wooden paddle. Above a certain drive level you should see the oscillations at the center become self-sustaining, while the water in the rest of the channel remains flat. This is a nonpropagating soliton, described in more detail in the paper of Wu *et al.* appended at the end. Map out the region of drive voltages and frequencies where the soliton is stable. Once one is started, it is possible to create more them along the channel with the paddle. Depending on their amplitudes and relative phase between them, they will either remain in place, merge with each other, or pass through each other as their position in the channel oscillates back and forth. A gradient in the height of the water along the channel will cause them to move towards the shallower end.

Analysis

A. From your sweeps, determine the resonant frequencies, and plot the phase velocity of each mode as a function of wavenumber. Compare to the theoretical predictions with and without the surface tension correction.

B. Determine the Q factor of the first 5-6 modes from the sweep data, and plot these versus wavenumber. From the time-decay data, determine values of α by plotting the ln of the amplitude versus time, and find the slope of the best-fit straight line. Convert these values to Q factors by using $Q = \omega_o/2\alpha$, and compare with the Q factors obtained from the sweep data.

C. Present and discuss your observations of the soliton properties.

D. As always, determine the uncertainties in your data, and display this as the error bars of plotted data. Discuss the factors that limit the accuracy of your results.