Physics 180D EXPERIMENT #4

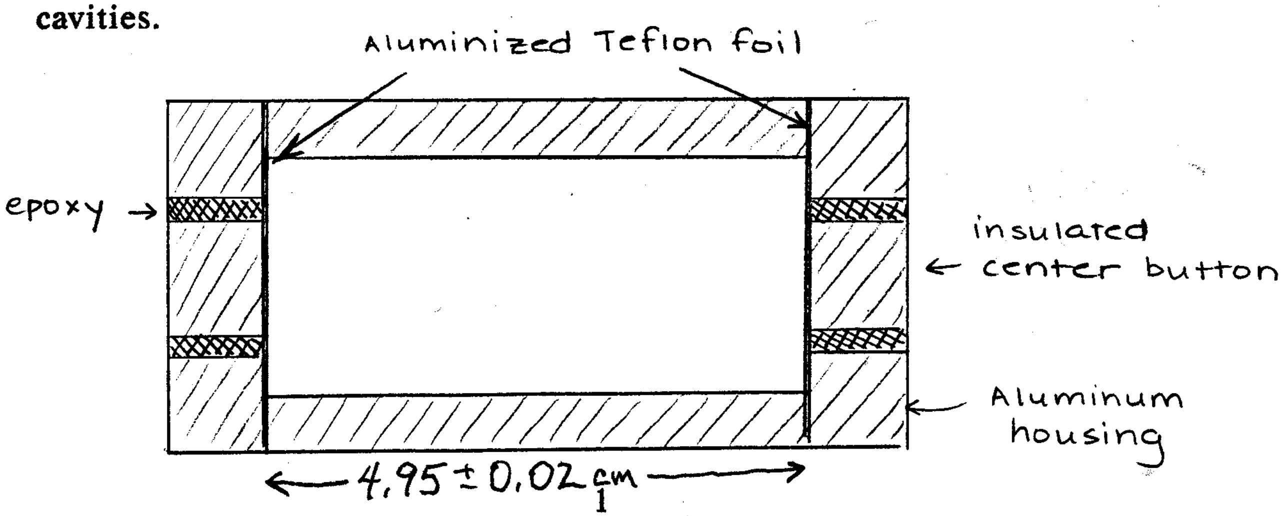
ACOUSTIC INVESTIGATION OF LIQUID HELIUM

OBJECT:

The purpose of this experiment is to illustrate the use of acoustics to investigate the properties of a material, in this case a liquid. Many substances show very diverse behavior acoustically. This behavior reflects the underlying phenomena and gives a clue as to its nature. At second order phase transitions, many times an anomalous attenuation and dispersion of sound occurs. In quantum fluids this diverse behavior is even more apparent. Helium 3 at low temperatures and high frequencies exhibits not ordinary sound but a collective excitation known as zero sound which is actually a distortion of the Fermi surface of this Fermi liquid. Helium 4 is a Bose liquid and displays superfluid properties, one of which is its ability to support wave motions unlike any other fluid. The aim of this experiment is to study three of these sounds and deduce other information about the fluid from them.

DESCRIPTION OF THE EXPERIMENT

The experiment consists essentially of measuring the phase velocity of 1st, 2nd and 4th sound through the plane wave resonant mode of cylindrical cavities.



A teflon electret foil is aluminized on the side opposite to the insulated center buttons. The aluminized side is connected to ground. The teflon foil and the insulated center button act as a capacitor microphone on one end and a capacitor loudspeaker on the other end.

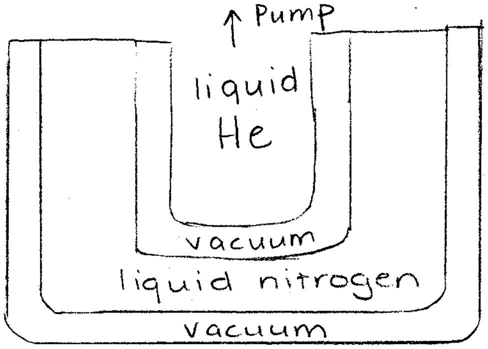
One can think of 1st sound as being ordinary sound that propagates in a liquid, our liquid being helium. In other words, energy is transported by changes in the density of the medium.

2nd sound is a "temperature" wave. The normal component of helium carries all the entropy of the fluid. So it is hotter than the superfluid component. The normal component flows toward cooler regions (like a normal liquid) but the superfluid component flows toward hotter regions. Waves can be set up by initially perturbing the system such that there is more superfluid in one region than another (the same goes for the normal component). Now the superfluid will flow to the region where there is more normal fluid and vice versa. This "counterflow" is known as 2nd sound and we will get this mode by piercing the teflon foil with many narrow slits made by a scalpel.

In the case of 4th sound the interior of the cavity is tightly packed with a fine powder. The porosity of the interior (defined as the percentage of space not occupied by the powder) is ~ 40%. In any ordinary fluid, of course, no sound could propagate in such a resonator. Thus this tightly packed medium is called a "superleak" because only superfluid helium can go through such narrow channels.

With these arrangements the measurement of the resonant frequencies of all the resonators gives the velocities of 1st, 2nd and 4th sound. The inside diameter of the resonators is 10mm and the length is 49,5 ± 0,2 mm

A standard cryostat system (shown below) is used to produce the low temperatures involved in this experiment.



There are 2 vacuum jackets. The outer one is to prevent heat leaks from the outside. Liquid nitrogen is placed in the outer dewar to cool the system down to 77K. A small amount of air is allowed into the inner vacuum jacket so there is some thermal contact between the outer and inner dewars. You want the inner dewar to be at liquid nitrogen temperatures so that liquid helium collects more efficiently.

PROCEDURE

1) Add liquid ^{4}He to the inner dewar. The helium is at room pressure and the temperature of the bath can be determined from thermodynamic tables.
2) Using a spectrum analyzer, drive the acoustic thermometer at resonance and determine which mode you're driving (1st, 2nd, 3rd, etc..).

3) Cool the dewar by slowly opening the pressure valve all the way.
4) While warming up (closing the pressure valve), measure f_{c1}, P gauge, and the resistance of the resistor thermometer, R_t. You should calibrate the resistor thermometer using the ^{4}He vapor pressure. Plot log R vs. T. Use this plot for the temperature measurements. Take data every 0.02K to about 2.20K.
5) Cool the dewar again. While warming up take measurements of f_{c2},f_{c4},P gauge, and R_t.

WRITE-UP

1) Plot c_1 vs. T.
2) Plot c_2 vs. T.
3) Because of the powder in the 4th sound cell there is a temperature independent scattering correction which results in a lower measured velocity than would be observed if the normal fluid component were locked by a powder which occupied a vanishingly small volume. Determine this scattering, *n*, by taking the ratio of c_4 in the attached thermodynamic table to the value of $f\lambda$ at or near your lowest temperature. Multiply all your observed values of $f\lambda$ by this number. This gives c_4. Plot c_4 vs. T.
4) An empirical result is that

$$n = \sqrt{2-P}$$

where P, the porosity, is equal to the helium volume divided by the helium volume and the powder volume combined. Determine P.
5) From the 4th sound velocity compute ρ_s/ρ and plot this as a function of temperature.
6) From the 2nd sound velocity and ρ_s/ρ compute the specific heat at constant pressure (Cp) and plot as a function of temperature. The values of the entropy are given in the accompanying table.
7) Discuss your results.

TABLE II. Density ρ, thermal expansion coefficient β_p, normal-fluid fraction ρ_n/ρ, entropy S, specific heat C_p, specific-heat ratio γ, isothermal compressibility κ_T, and first-, second-, and fourth-sound velocities (C_1, C_2, C_4) at various temperatures and pressures (T, P) in the He II phase. The entries are self-consistent in that thermodynamic identities [e.g. $C_p = T(\partial S/\partial T)_P$, $\beta_p = -\rho(\partial S/\partial P)_T$] and the sound-velocity equations are obeyed exactly. The functions which generated the tables had the correct asymptotic behavior as $T \to T_\lambda$ built in.

T (K)	ρ (g cm^{-3})	$-10^3\beta_p$ (K^{-1})	ρ_n/ρ	S (J g^{-1} K^{-1})	C_p (J g^{-1} K^{-1})	$10^2(\gamma-1)$	$10^2\kappa_T$ (bar^{-1})	C_1 (m sec^{-1})	C_2 (m sec^{-1})	C_4 (m sec^{-1})
					$P=0.0$					
1.20	0.1452	0.02	0.0283	0.0515	0.318	0.000	1.222	237.4	18.55	234.0
1.25	0.1452	0.42	0.0368	0.0663	0.408	0.003	1.225	237.1	18.78	232.8
1.30	0.1452	0.78	0.0472	0.0843	0.515	0.009	1.228	236.8	19.03	231.2
1.35	0.1452	1.11	0.0598	0.1060	0.641	0.015	1.231	236.5	19.30	229.4
1.40	0.1453	1.45	0.0748	0.1319	0.786	0.021	1.236	236.1	19.58	227.2
1.45	0.1453	1.84	0.0924	0.1623	0.953	0.029	1.240	235.6	19.84	224.7
1.50	0.1453	2.32	0.1131	0.1978	1.142	0.039	1.245	235.2	20.07	221.7
1.55	0.1453	2.91	0.1369	0.2387	1.357	0.053	1.251	234.6	20.25	218.3
1.60	0.1453	3.62	0.1643	0.2855	1.598	0.072	1.257	234.0	20.37	214.3
1.65	0.1453	4.44	0.1955	0.3387	1.869	0.095	1.264	233.4	20.41	209.8
1.70	0.1454	5.30	0.2310	0.3990	2.174	0.119	1.272	232.7	20.36	204.6
1.75	0.1454	6.17	0.2712	0.4668	2.514	0.142	1.281	231.9	20.19	198.6
1.80	0.1454	7.00	0.3166	0.5429	2.896	0.162	1.291	231.0	19.89	191.7
1.85	0.1455	7.79	0.3677	0.6279	3.323	0.178	1.302	230.0	19.43	183.7
1.90	0.1455	8.63	0.4253	0.7228	3.804	0.195	1.315	228.8	18.78	174.4
1.95	0.1456	9.79	0.4901	0.8285	4.351	0.222	1.331	227.4	17.89	163.5
2.00	0.1457	11.72	0.5632	0.9465	4.990	0.281	1.350	225.8	16.69	150.4
2.05	0.1458	15.09	0.6469	1.0791	5.791	0.405	1.373	224.0	15.00	134.4
2.10	0.1459	19.61	0.7484	1.2316	6.972	0.570	1.401	221.8	12.39	112.6
2.15	0.1460	22.85	0.8856	1.4162	8.723	0.619	1.434	219.2	7.99	75.1
					$P=1.0$					
1.20	0.1469	0.44	0.0291	0.0517	0.325	0.004	1.133	245.0	18.14	241.5
1.25	0.1469	0.88	0.0380	0.0667	0.415	0.014	1.135	244.8	18.41	240.2
1.30	0.1470	1.34	0.0489	0.0850	0.523	0.027	1.137	244.6	18.70	238.7
1.35	0.1470	1.83	0.0620	0.1071	0.649	0.042	1.140	244.4	19.00	236.8
1.40	0.1470	2.35	0.0775	0.1332	0.795	0.058	1.143	244.0	19.28	234.6
1.45	0.1470	2.93	0.0958	0.1640	0.964	0.077	1.147	243.7	19.55	232.0
1.50	0.1470	3.58	0.1170	0.1999	1.155	0.099	1.151	243.2	19.78	228.9
1.55	0.1470	4.33	0.1416	0.2412	1.372	0.125	1.156	242.7	19.96	225.3
1.60	0.1471	5.18	0.1698	0.2886	1.617	0.156	1.161	242.2	20.07	221.2
1.65	0.1471	6.12	0.2020	0.3424	1.890	0.191	1.167	241.6	20.11	216.4
1.70	0.1472	7.15	0.2386	0.4033	2.196	0.230	1.174	240.9	20.05	210.9
1.75	0.1472	8.25	0.2799	0.4718	2.537	0.270	1.182	240.1	19.87	204.5
1.80	0.1473	9.35	0.3266	0.5485	2.920	0.308	1.191	239.2	19.56	197.2
1.85	0.1473	10.53	0.3790	0.6343	3.349	0.348	1.201	238.2	19.08	188.7
1.90	0.1474	11.91	0.4379	0.7299	3.836	0.394	1.213	237.0	18.40	178.8
1.95	0.1475	13.70	0.5040	0.8366	4.399	0.462	1.227	235.6	17.47	167.2
2.00	0.1476	16.38	0.5787	0.9562	5.071	0.580	1.244	234.0	16.20	153.3
2.05	0.1477	20.70	0.6644	1.0914	5.927	0.799	1.266	232.2	14.43	136.1
2.10	0.1478	27.65	0.7679	1.2480	7.161	1.186	1.294	230.0	11.75	112.4
2.15	0.1480	41.51	0.9093	1.4387	9.313	2.056	1.334	227.4	6.91	69.7
					$P=2.0$					
1.20	0.1486	0.78	0.0302	0.0521	0.332	0.014	1.058	252.3	17.76	248.5
1.25	0.1486	1.25	0.0395	0.0674	0.423	0.029	1.059	252.1	18.07	247.2
1.30	0.1486	1.76	0.0509	0.0861	0.532	0.048	1.061	252.0	18.39	245.6
1.35	0.1486	2.31	0.0645	0.1085	0.659	0.070	1.062	251.8	18.70	243.7
1.40	0.1486	2.91	0.0806	0.1350	0.807	0.093	1.065	251.5	19.00	241.4
1.45	0.1486	3.57	0.0995	0.1662	0.976	0.119	1.068	251.2	19.27	238.6
1.50	0.1487	4.30	0.1214	0.2025	1.170	0.149	1.071	250.8	19.51	235.4
1.55	0.1487	5.11	0.1468	0.2444	1.390	0.183	1.075	250.3	19.68	231.7
1.60	0.1487	6.02	0.1758	0.2924	1.637	0.221	1.080	249.8	19.79	227.3
1.65	0.1488	7.04	0.2090	0.3469	1.913	0.265	1.085	249.2	19.82	222.3

The density, expansion coefficient, saturated vapour pressure, specific heat, and entropy of liquid ^{4}He under the saturated vapour pressure

Density ρ Expansion coefficient α		Kerr and Taylor [1].
Vapour pressure P		Van Dijk and Durieux [2].
Specific heat C	below 0·7°	Wiebes *et al.* [3].
Entropy S	0·7° to 1·8°	Kramers *et al.* [4].
	1·8° to 4·4°	Hill and Lounasmaa [5].

T(°K)	ρ (g cm^{-3})	α (10^{-3} deg^{-1})	P (cm Hg)	C (Jg^{-1}deg^{-1})	S (Jg^{-1}deg^{-1})
0·2	0·1450	+0·009		0·0002	0·00005
0·3	0·1450	+0·028		0·0005	0·00018
0·4	0·1450	+0·068		0·0013	0·00044
0·5	0·1450	+0·134	$1{\cdot}634 \times 10^{-6}$	0·0025	0·00085
0·6	0·1450	+0·227	$2{\cdot}812 \times 10^{-5}$	0·0044	0·00147
0·7	0·1450	+0·343	$2{\cdot}279 \times 10^{-4}$	0·0093	0·00276
0·8	0·1450	+0·461	$1{\cdot}145 \times 10^{-3}$	0·0222	0·00475
0·9	0·1450	+0·514	$4{\cdot}158 \times 10^{-3}$	0·0510	0·00885
1·0	0·1450	+0·465	$1{\cdot}200 \times 10^{-2}$	0·1042	0·0168
1·1	0·1450	+0·261	$2{\cdot}922 \times 10^{-2}$	0·191	0·0304
1·2	0·1450	−0·137	$6{\cdot}250 \times 10^{-2}$	0·322	0·0523
1·3	0·1450	−0·754	$1{\cdot}209 \times 10^{-1}$	0·516	0·0853
1·4	0·1450	−1·61	$2{\cdot}155 \times 10^{-1}$	0·780	0·132
1·5	0·1451	−2·72	$3{\cdot}599 \times 10^{-1}$	1·127	0·197
1·6	0·1451	−4·03	$5{\cdot}690 \times 10^{-1}$	1·572	0·284
1·7	0·1452	−5·71	$8{\cdot}590 \times 10^{-1}$	2·11	0·395
1·8	0·1453	−7·59	1·247	2·81	0·535
1·9	0·1454	−9·98	1·748	3·79	0·715
2·0	0·1456	−13·2	2·377	5·18	0·940
2·1	0·1458	−18·4	3·143	7·51	1·24
2·2	0·1460	+11·71	4·047	3·98	1·61
2·3	0·1457	+27·0	5·101	2·64	1·74
2·4	0·1453	+32·3	6·330	2·38	1·85
2·5	0·1448	+36·9	7·749	2·28	1·93
2·6	0·1442	+42·0	9·378	2·27	2·02
2·8	0·1429	+52·8	13·295	2·34	2·19
3·0	0·1412	+64·6	18·207	2·49	2·36
3·2	0·1392	+76·9	24·227	2·69	2·53
3·4	0·1369	+89·1	31·470	2·97	2·71
3·6	0·1343	+100·9	40·047	3·26	2·89
3·8	0·1315	+112·7	50·069	3·60	3·06
4·0	0·1284	+124·6	61·654	3·99	3·26
4·2	0·1251	+137·3	74·933	4·48	3·46
4·4	0·1216	+151·0	90·026	5·11	3·68

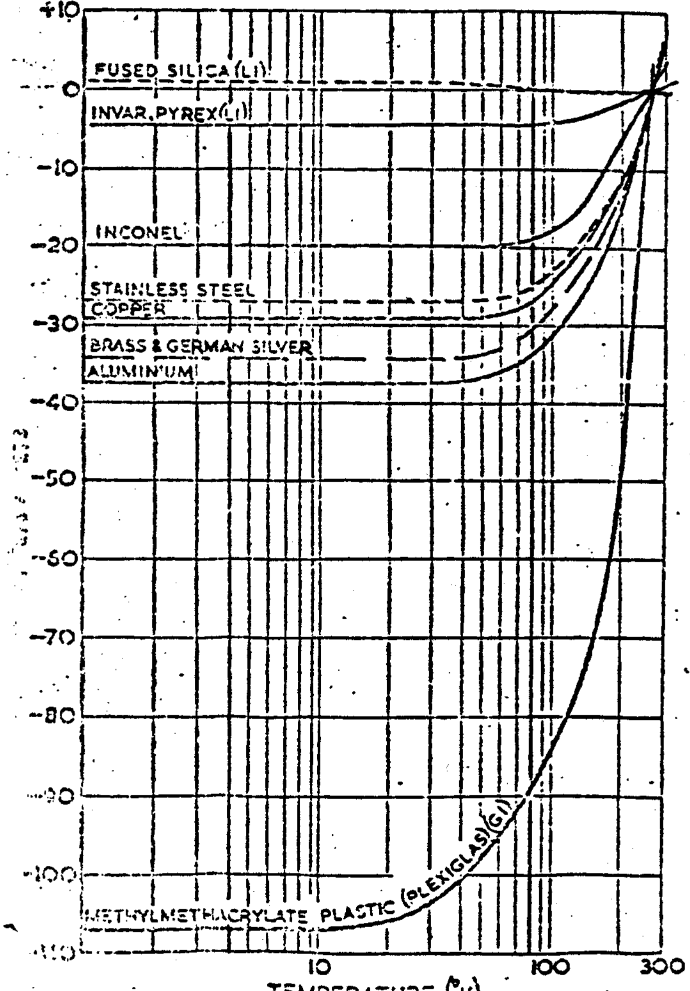
The velocity of second sound [10], the ratio ρ_n/ρ, and the viscosity η_n [11] of liquid helium II under the saturated vapour pressure

T(°K)	c_2 (m sec^{-1})	ρ_n/ρ	η_n (micropoise)
0·6	83·0	$4{\cdot}27 \times 10^{-5}$	
0·7	46·5	$2{\cdot}52 \times 10^{-4}$	
0·8	29·0	$9{\cdot}66 \times 10^{-4}$	162
0·9	21·6	$2{\cdot}95 \times 10^{-3}$	65·4
1·0	18·9	$7{\cdot}52 \times 10^{-3}$	35·1
1·1	18·3	$1{\cdot}56 \times 10^{-2}$	23·2
1·2	18·4	$2{\cdot}92 \times 10^{-2}$	18·2
1·3	19·1	$4{\cdot}78 \times 10^{-2}$	16·1
1·4	19·7	$7{\cdot}54 \times 10^{-2}$	15·1
1·5	20·1	0·11	14·2
1·6	20·3	0·17	13·3
1·7	20·3	0·24	13·0
1·8	19·9	0·32	13·0
1·9	18·7	0·43	13·6
2·0	16·4	0·56	15·0
2·1	12·2	0·74	18·6

Tables are from

WILKS LIQUID AND SOLID HELIUM

c = [illegible](1.2)E10 cm/sec
e = 1.602[illegible]92(46)E-19 coul
4.803242(19)E-10 esu
h = 6.626196E-27 erg sec
ℏ = 1.0545887(57)E-27 erg sec
ℏc = 197.32858(51) Mev fm
N = 6.022045(31)E23 mole-1
Me = 9.109534(47)E-28 gram
0.511034(14) Mev
Mp = 1.672614E-24 gram
938.2796(27) Mev
1836.15152(70) Me
K = 1.380622(44)E-16 erg K-1
8.61735(28)E-5 ev K-1
1 ev/ 11604.50(36) K
a0 = 0.52917706 A
e2/a0 = 27.21164661 ev
amu = 1.660531E-24 gram
M4 = 6.6464E-24 gram
M3 = 5.0097E-24 gram
(gas/liq)4 = 700.3
(gas/liq)3 = 608.0
Rho4 = 0.1451 g/cm3
1 mole/27.6 cm3
Rho~~4~~3 = 0.08187 g/cm3
1 mole/36.8 cm3
K^4 = 1.0572
alpha4 = 0.205 A3
e0 = 8.8541878 pF/m



THERMAL EXPANSION

TABLE 4k-5. VAPOR PRESSURE OF HELIUM 4 (1958 SCALE)

Vapor pressure of ⁴He. Unit 10^{-3} mm Hg at 0°C, g = 980.665 cm/sec²

T	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.5	.016342	.022745	.031287	.042561	.057292	.076356	.10081	.13190	.17112	.22021
	.28121	.35649	.44877	.56118	.69729	.86116	1.0574	1.2911	1.5682	1.8949
	2.2787	2.7272	3.2494	3.8549	4.5543	5.3591	6.2820	7.3365	8.5376	9.9013
	11.445	13.187	15.147	17.348	19.811	22.561	25.624	29.027	32.800	36.974
	41.581	46.656	52.234	58.355	65.059	72.386	80.382	89.093	98.567	108.853
1.0	120.000	132.070	145.116	159.198	174.375	190.711	208.274	227.132	247.350	269.006
	292.169	316.923	343.341	371.512	401.514	433.437	467.365	503.396	541.617	582.129
	625.025	670.411	718.386	769.057	822.527	878.916	938.330	1000.87	1066.67	1135.85
	1208.51	1284.81	1364.83	1448.73	1536.61	1628.62	1724.91	1825.58	1930.79	2040.67
	2155.35	2274.99	2399.73	2529.72	2665.09	2805.99	2952.60	3105.04	3263.48	3428.07
1.5	3598.97	3776.32	3960.32	4151.07	4348.79	4553.58	4765.68	4985.18	5212.26	5447.11
1.6	5689.88	5940.76	6199.90	6467.42	6743.57	7028.47	7322.31	7625.21	7937.40	8259.02
1.7	8590.22	8931.18	9282.06	9643.02	10014.3	10395.9	10788.2	11191.2	11605.1	12030.1
1.8	12466.1	12913.7	13372.8	13843.6	14326.1	14820.7	15327.3	15846.3	16377.7	16921.7
1.7	17478.2	18047.7	18630.1	19225.5	19834.1	20455.9	21091.1	21739.7	22402.0	23077.9
2.0	23767.4	24470.9	25188.1	25919.2	26664.2	27423.3	28196.3	28983.2	29784.2	30599.1
	31428.1	32271.1	33128.0	33998.6	34882.8	35780.3	36690.9	37614.8	38550.2	39500.3
	40465.6	41446.6	42443.5	43456.5	44485.7	45531.3	46593.5	47672.5	48768.6	49881.8
	51012.3	52160.2	53325.8	54509.2	55710.5	56930.0	58167.8	59423.8	60698.8	61992.0
	63304.3	64635.2	65985.4	67354.8	68743.5	70152.0	71580.2	73028.1	74496.0	75984.2
2.5	77493.1	79022.2	80572.2	82142.9	83734.6	85347.2	86981.2	88636.7	90313.8	92012.6
	93733.4	95476.0	97240.8	99028.2	100838	102669	104525	106403	108304	110228
	112175	114145	116139	118156	120198	122263	124353	126465	128603	130765
	132952	135164	137401	139663	141949	144260	146597	148961	151349	153763
	156204	158671	161164	163684	166230	168802	171402	174028	176682	179364
3.0	182073	184810	187574	190366	193187	196037	198914	201820	204755	207719
	210711	213732	216783	219864	222975	226115	229285	232484	235714	238974
	242266	245587	248939	252322	255736	259182	262658	266166	269706	273278
	276880	280516	284183	287883	291615	295380	299178	303008	306871	310768
	314697	318659	322654	326684	330747	334845	338976	343141	347341	351575
3.5	355844	360147	364485	368860	373269	377714	382194	386710	391262	395849
	400471	405130	409825	414556	419324	424128	428968	433846	438760	443713
	448702	453729	458794	463897	469038	474218	479435	484691	489985	495317
	500688	506098	511547	517036	522564	528132	533739	539387	545075	550805
	556574	562383	568234	574126	580059	586034	592051	598110	604210	610352
4.0	616537	622764	629033	635345	641700	648099	654541	661026	667554	674125
	680740	687399	694103	700851	707643	714479	721360	728285	735255	742269
	749328	756431	763579	770772	778010	785294	792023	799999	807422	814893
	822411	829978	837592	845255	852966	860725	868533	876390	884296	892252
	900258	908313	916418	924573	932778	941033	949338	957693	966099	974556
4.5	983066	991628	1000239	1008905	1017621	1026390	1035213	1044087	1053014	1061995
	1071029	1080114	1089254	1098449	1107699	1117002	1126359	1135772	1145239	1154761
	1164339	1173972	1183662	1193407	1203209	1213066	1222981	1232955	1242983	1253069
	1263212	1273414	1283673	1293991	~~[illegible]~~	1314802	1325297	1335850	1346462	1357136
	1367870	1378662	1389516	1400429	1411404	1422438	1433533	1444690	1455911	1467191
5.0	1478535	1489940	1501409	1512940	1524535	1536192	1547912	1559698	1571546	1583458
	1595437	1607481	1619589	1631761	1644000	1656305	1668673	1681108	1693612	1706180
	1718817	1731521	1744290							

Thermal Expansion 1E4*(L(t) - L(273)) / L (273)

Element	4K	77K
Copper	-29.3	-27.2
Aluminum	-37.4	-34.8
Nickel	-20.4	-19.5
Brass	-34.5	-31.6
German Silver	-33.9	-31.3
302 Stainless Steel	-28.4	-26.9
304,316 Stainless	-26.3	-25.2
Invar	- 4.6	- 4.6
Monel	-22.4	-21.1
Cupro-Nickel (70/30)	-25.2	-23.8
Inconel	-20.4	-19.4

Attachment for Exp 5

Introduction to HeII

2.1. General introduction

By now there are many fine books on the subject of superfluid helium. Among the most recent and/or best known are those by Atkins, Donnelly, Khalatnikov, London, Putterman, and Wilks (see the references for a complete listing of each). For a general introduction to superfluid helium one of them should be consulted. I will only introduce that which I consider useful for the understanding of sound propagation in a porous solid saturated with superfluid helium. Also, it is to be understood that by helium I will always mean the common isotope, ^{4}He.

Liquid helium exists in two phases, separated by a phase transition: the so called lambda transition. Figure 2.1 shows its phase diagram. The lambda transition is indicated by the near vertical line in the middle of the figure. It occurs at 2.172K at saturated vapor pressure (SVP). At temperatures above the lambda line the liquid is called HeI. At temperatures below the lambda line the liquid is called HeII.

Liquid HeI is an ordinary liquid in every sense -- it has a small but finite viscosity and carries entropy with it as it flows. Classical hydrodynamics provides an excellent description of HeI.

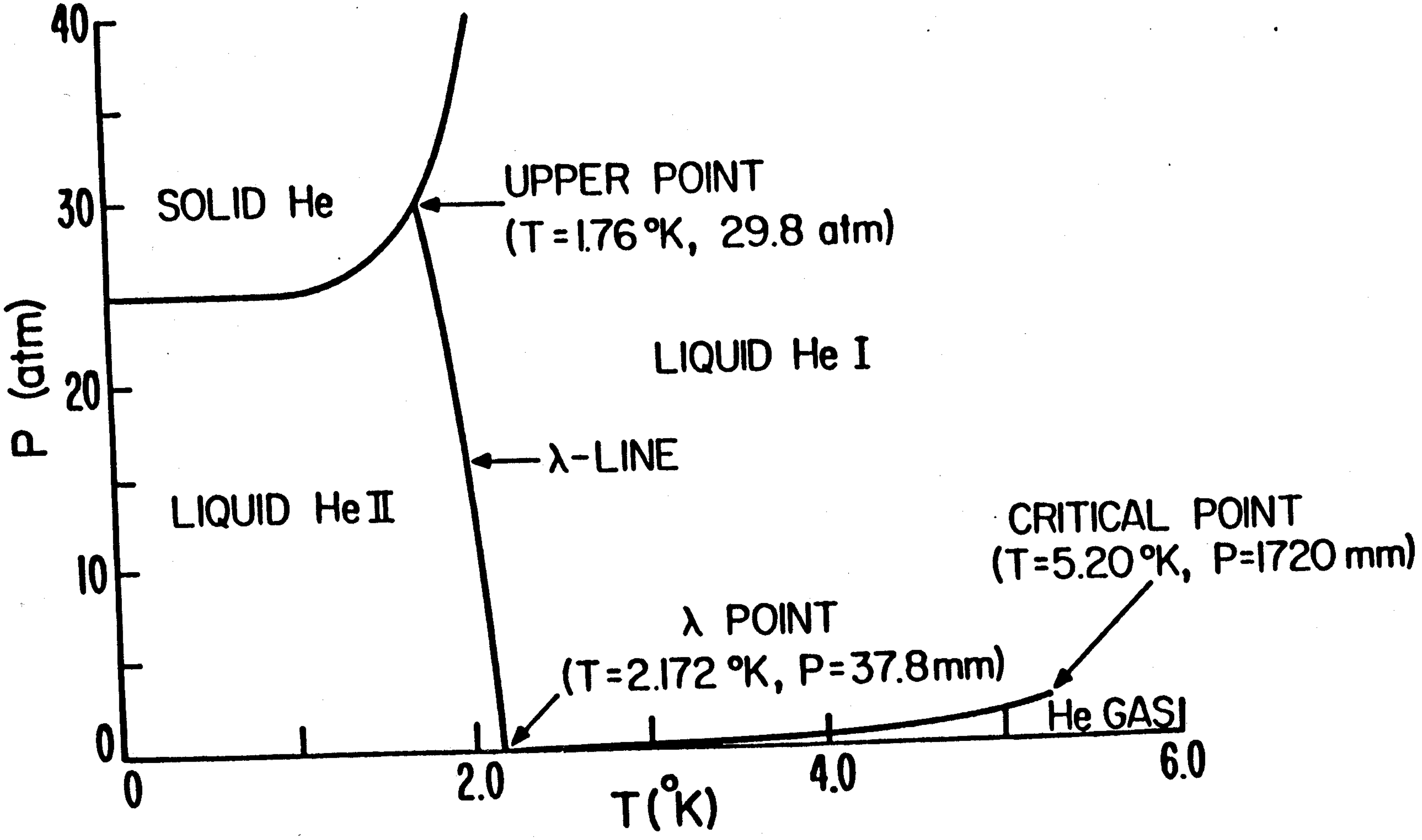


Figure 2.1. The phase diagram of liquid helium.

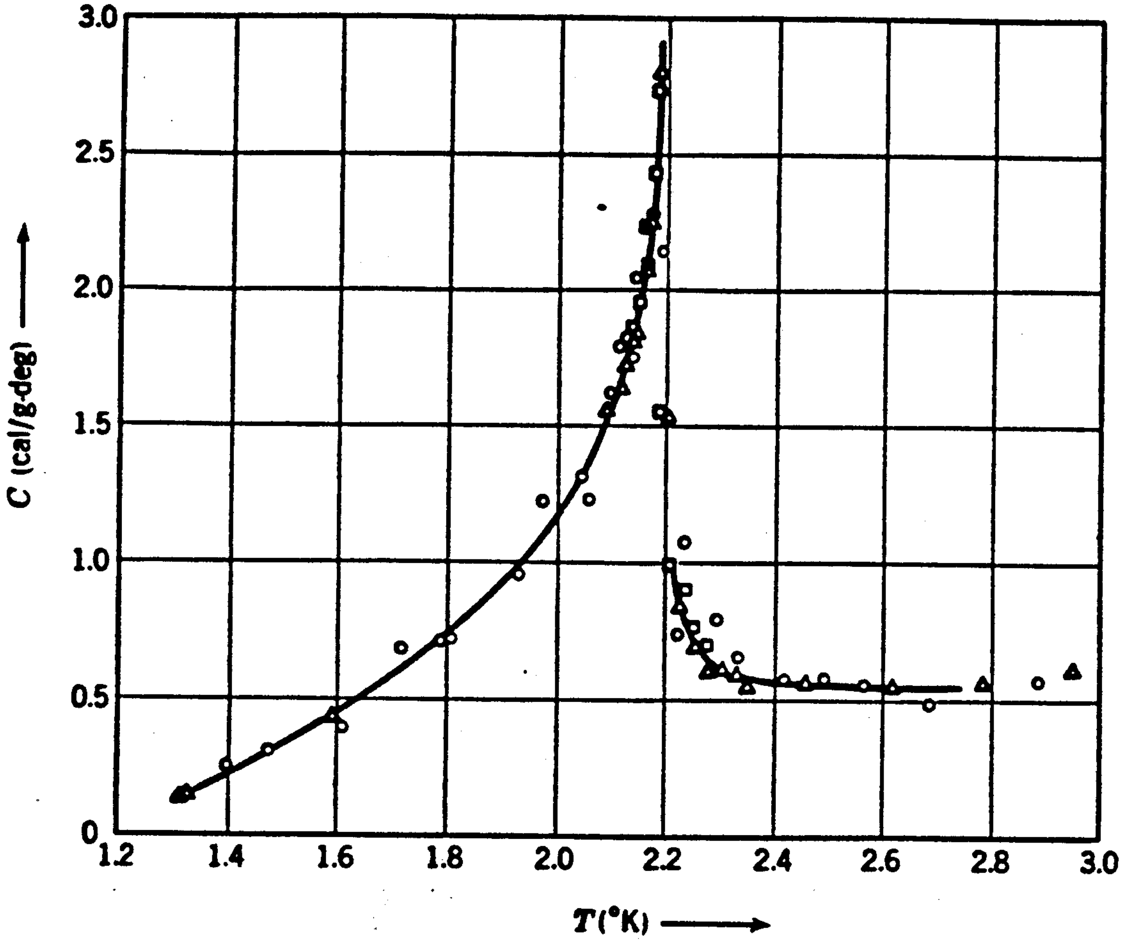


Fig. 2. Specific heat of liquid helium under its own vapor pressure (after Keesom and Clusius[4] and Keesom and Keesom[4]).

helium presents the unique case of a substance which cannot be solidified under its own vapor pressure merely by cooling.

It might be argued that we can never know with certainty what might happen if still lower temperatures were reached. However, it appears very improbable that solidification will occur no matter how closely we might approach absolute zero. At temperatures below 1°K the melting-pressure curve $P_m(T)$ is practically horizontal. From the Clausius-Clapeyron equation,

$$dP_m/dT = (S_{\text{liquid}} - S_{\text{solid}})/(V_{\text{liquid}} - V_{\text{solid}}) \tag{1}$$

it follows that at 1°K the entropy of the liquid must be practically equal to the entropy of the solid. It looks therefore as if liquid helium II at about 1°K has fulfilled the requirements of Nernst's law, at least as well as solid helium does at the same temperature. Apparently the liquid could not gain with respect to thermal order by solidifying at a still lower temperature. Figure 3 shows that there is a sudden turn in the melting curve $P_m(T)$ toward its horizontal course at lowest temperatures. The temperature at which this turn occurs coincides with the region of *anomalously high specific heat* close to the λ-line. Hence it

Liquid HeII, on the other hand, is like no classical liquid. It behaves as though there were two fluids present, one of which is ordinary, just like HeI, and another which flows without viscosity and carries no entropy. The two fluid model of HeII was constructed to explain this unusual behavior.

2.2. Two fluid model of HeII

The two fluid model supposes that liquid HeII is composed of two independent, interpenetrating fluids. The two component fluids are referred to as the normal component, or normalfluid, and the superfluid component, or superfluid. The latter got its name from its ability to flow seemingly without friction (which in fact it does). The superfluid component is actually a macroscopic occupation of the quantum mechanical many body ground state of the liquid. This is what accounts fo- its unusual properties (i.e. zero viscosity, zero entropy), and is why liquid HeII is called a quantum fluid.

The fractional amount of normalfluid and superfluid present in HeII are thermodynamic state functions, and so they depend on temperature and pressure. Figure 2.2 shows the fractional amount of each as a function of temperature at SVP. At T_λ (and above) HeII is entirely normalfluid, but by 2K the amount of superfluid and normalfluid is approximately equal. By 1K it is about 99% superfluid. At 0K HeII is entirely superfluid.

The normalfluid and superfluid each have their own mass density

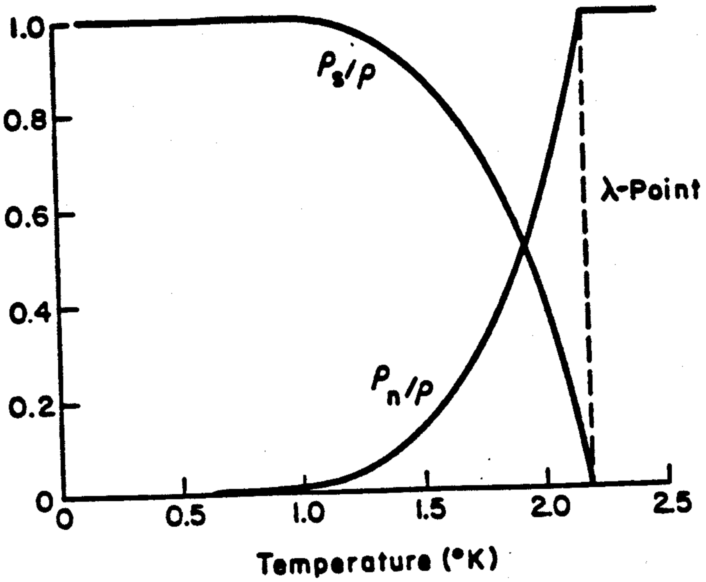


Figure 2.2. ρ_s/ρ and ρ_n/ρ vs. T at SVP.

Classical Fluid		HeII		
		Superfluid	Normalfluid	
ρ	density	ρ_s	ρ_n	$\rho = \rho_s + \rho_n$
$\vec{v}$	velocity	$\vec{v}_s$	$\vec{v}_n$	$\rho\vec{v}_{cm} = \rho_s\vec{v}_s + \rho_n\vec{v}_n$
η	viscosity	0	η	
s	entropy	0	s	

Table 2.1. Classical fluid vs. two-fluid model of HeII.

and velocity field, labeled with the subscripts n and s for normalfluid and superfluid, respectively:

ρ_n = normalfluid mass density,
ρ_s = superfluid mass density,
v_n = normalfluid velocity,
v_s = superfluid velocity.

The total fluid mass density, ρ, is the sum of the two mass densities:

$$\rho_n + \rho_s = \rho = \text{total fluid mass density.}$$

The mass current, or momentum density, of each fluid is the mass density times the velocity:

$\rho_n v_n$ = normalfluid momentum density,
$\rho_s v_s$ = superfluid momemtum density.

The center of mass velocity, v_{cm}, is implicitly defined by the total momentum density of the fluid:

$$\rho_n v_n + \rho_s v_s = \rho v_{cm} = \text{total fluid momentum density.}$$

Table 2.1 compares the features of the two fluid model of HeII with a classical fluid.

2.2.1. Two fluid equations

The Landau two fluid equations form the basis of the dynamical description of HeII [Landau 1941, Putterman, Ch. 1]. They are based on the results of early experiments on superfluid helium. In linearized form they are

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\rho\mathbf{v}_{cm} = 0 \qquad \rho\mathbf{v}_{cm} = \rho_s\mathbf{v}_s + \rho_n\mathbf{v}_n \tag{2.2.1}$$

$$\frac{\partial\rho s}{\partial t} + \nabla\cdot\rho s\mathbf{v}_n = 0 \tag{2.2.2}$$

$$\rho\frac{\partial\mathbf{v}_{cm}}{\partial t} = -\nabla p \tag{2.2.3}$$

$$\rho_s\frac{\partial\mathbf{v}_s}{\partial t} = -\rho_s\nabla\mu \qquad d\mu = -s\,dT + \frac{1}{\rho}dp$$

$$= -\frac{\rho_s}{\rho}\nabla p + \rho_s s\nabla T \tag{2.2.4}$$

Note that mass convects with the center of mass velocity, $\mathbf{v}_{cm}$ (Eq. (2.2.1)), but that entropy convects with the normalfluid velocity, $\mathbf{v}_n$ (Eq. (2.2.2)), since the superfluid has none. Also note that the center of mass responds to gradients in pressure, the same as an ordinary fluid, but the superfluid component responds only to gradients in the chemical potential, μ.

Equations (2.2.3) and (2.2.4) can be combined to yield an equation of motion for the normalfluid component:

$$\rho_n \frac{\partial v_n}{\partial t} = -\nabla(p - \rho_s \mu)$$

$$= -\frac{\rho_n}{\rho}\nabla p - \rho_s s \nabla T \tag{2.2.5}$$

Note that both the normalfluid and superfluid components respond to gradients in pressure and temperature, but whereas they both flow away from regions of higher pressure, the superfluid flows toward regions of higher temperature where the normalfluid flows away. This has interesting consequences for sound propagation in HeII.

The Landau equations must be supplemented with the restriction that the superfluid velocity field be curl free, i.e.

$$\nabla \times v_s = 0, \tag{2.2.6}$$

[Putterman, pp. 102-105, Putterman 1982]. This restriction leads to interesting consequences, such as quantized superfluid vortices.

2.3. Sound modes in bulk HeII

The propagating sound modes in bulk HeII are derived from the linearized two fluid equations by inserting plane wave expressions for all the first order quantities [Putterman, p. 44]. When the normalfluid is free to move the propagating sound modes are first sound and second sound. When the normalfluid is immobilized the

Table 2.2. The propagating sound modes in bulk HeII.

SOUND MODE	SUPERFLUID VELOCITY	NORMALFLUID VELOCITY	TYPE OF WAVE	WAVE SPEED
1st	→	→	PRESSURE DENSITY	$c_1^2 = \left(\frac{\partial p}{\partial \rho}\right)_s$
2nd	→	←	TEMPERATURE ENTROPY	$c_2^2 = \frac{\rho_s}{\rho_n}\frac{s^2 T}{c_p}$
4th	→	ZERO	PRESSURE WAVE IN SUPERLEAK	$c_4^2 = \frac{\rho_s}{\rho} c_1^2$
5th	→	ZERO	THERMAL WAVE IN SUPERLEAK	$c_5^2 = \frac{\rho_s}{\rho}\frac{s^2 T}{c_p}$

useful: $\frac{\rho_s}{\rho_n} = \frac{\rho_s}{\rho - \rho_s} = \frac{\rho_s/\rho}{1 - \rho_s/\rho}$

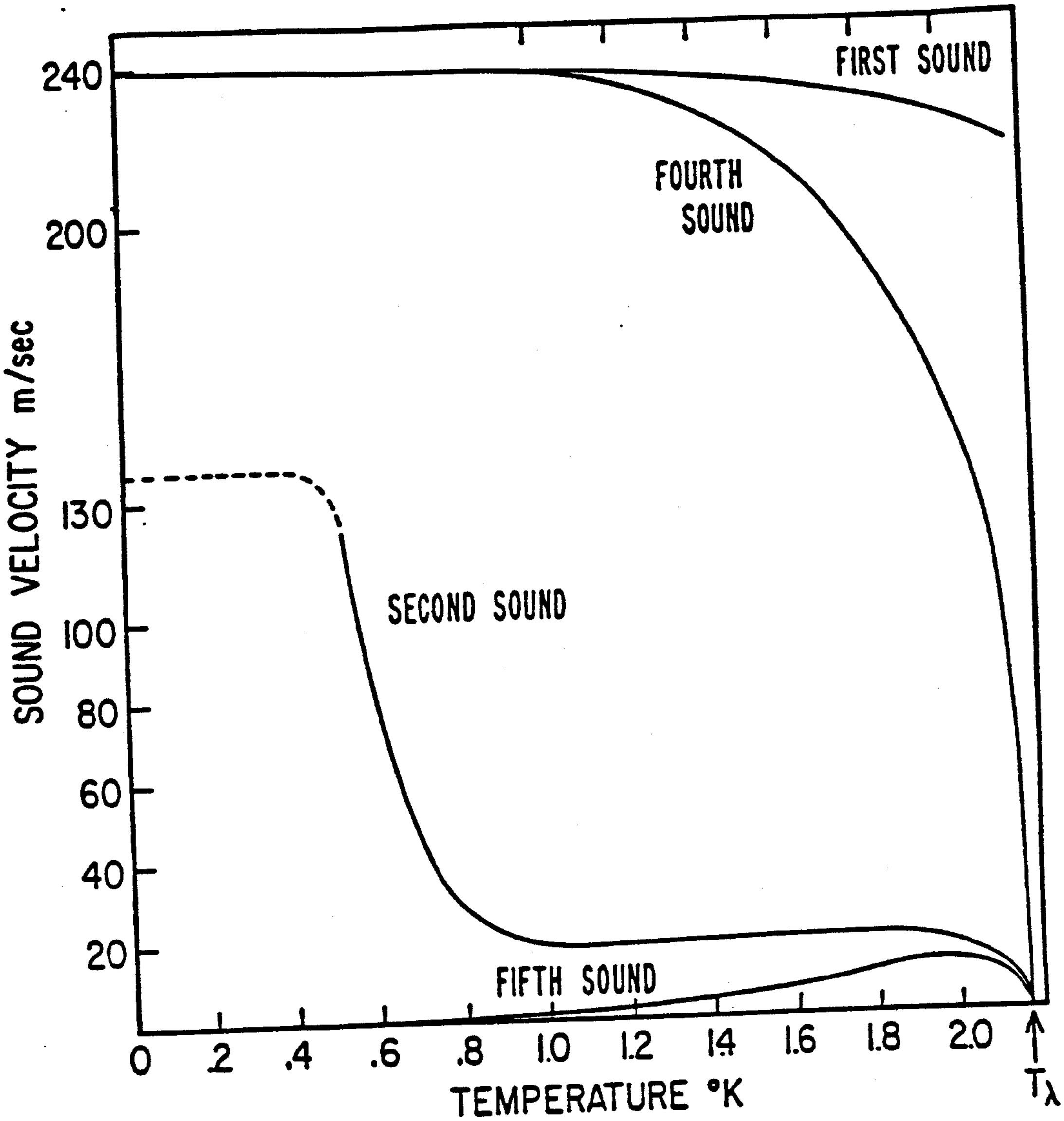


Figure 2.3. Speeds of the propagating sound modes in bulk HeII vs. T at SVP.

propagating sound modes are fourth sound and fifth sound. Table 2.2 gives approximate expressions for the speeds of these modes and some of their characteristic features. Figure 2.3 shows their speeds as a function of temperature at SVP. A brief description of each sound mode is given below.

First sound

First sound is a pressure-density wave. It is completely analagous to ordinary sound in a classical fluid. The normalfluid and superfluid move in phase with each other in first sound with essentially the same velocity, the velocity of the center of mass, v_{cm}. What little temperature swings exist are largely due to the adiabatic compressions and rarefactions accompanying the concentration and dilution of mass. First sound is produced and received by vibrating piston type transducers, the same as ordinary sound.

Second sound

Second sound is a propagating temperature-entropy wave -- it has no counterpart in a classical fluid (in superfluid helium heat flows reversibly!). In second sound the normalfluid and superfluid move out of phase with each other in such a way that the center of mass remains essentially fixed, so there are practically no density (pressure) swings. This counterflow leads to the separation of superfluid and normalfluid which in turn results in the concentration and dilution of entropy, and therefore to hot and cold spots, which drives the wave. The wave is sustained by the inertia of each fluid component.

Any transducer that produces counterflow of superfluid and normalfluid will generate second sound. Common sources include a resistive heater and a diaphragm perforated with holes so small that only the superfluid can pass through them (called an oscillating superleak transducer) [Williams et. al. 1969, Sherlock and Edwards 1970]. Either a sensitive thermometer (usually made from a carbon resistor) or another perforated diaphragm can be used to receive second sound. The diaphragm transducers have the advantage over the resistive heater/thermometer transducers that they do not introduce any unwanted steady heat into the fluid. Another advantage of the diaphragm transducers is that they can also be used to generate and receive first sound, so that it is possible to observe both first sound and second sound in the same experimental cell.

Fourth sound

Fourth sound is what first sound becomes when the normalfluid is immobilized, while the superfluid remains free to move. This is reflected in the factor of (ρ_s/ρ) in the expression for the square of its wave speed in Table 2.2. Like second sound, fourth sound is unique to HeII. It is a thermomechanical wave, with contributions to the restoring force from both pressure and temperature swings, although the restoring force in response to pressure swings is considerably larger. Hence I have neglected the thermal contribution in the expression for the square of its wave speed in Table 2.2. Fourth sound is driven and received with the same sort of transducers as first sound.

Fifth sound [Williams et. al. 1979, Jelatis et. al. 1979]

Fifth sound is what fourth sound becomes when pressure swings are not allowed, i.e. when the restoring force is due entirely to the temperature swings associated with the separation of superfluid and normalfluid. It too is unique to HeII. The expression for the square of the speed of fifth sound is the term in the expression for the square of the speed of fourth sound that is usually neglected. Hence the speed of fifth sound is considerably smaller than the speed of fourth sound (or first sound). For practical reasons fifth sound is usually generated and received with thermal transducers.

The counterpart to second sound when the normalfluid is immobilized is a nonpropagating (i.e. diffusive) thermomechanical wave, and is not included among the bulk sound modes for this reason.

2.4. Scattering correction

In order to observe fourth sound in HeII the normalfluid must be immobilized. This is accomplished routinely in the laboratory by introducing HeII into a porous solid whose pores are so small (typically 1 micron or smaller) that the viscous drag of the microscopic channel walls on the normalfluid essentially locks it to the solid. Such a porous solid is called a superleak.

Superleaks are commonly made by packing fine powder (such as abrasive powder) into a mold. However, the presence of powder grains causes the fourth sound wave to be multiply scattered. As a result, the experimentally observed wave speed is always less than predicted by the Landau theory. The factor by which the observed speed of fourth sound is reduced is found to be a constant for a particular superleak, and is called the fourth sound scattering correction, or index of refraction, n, of the superleak:

$$c_{4,obs} = \frac{c_{4,th}}{n}. \qquad (2.4.1)$$

The value of the scattering correction varies from superleak to superleak, depending most strongly on the fraction of the total volume occupied by the porous solid: the more solid the greater the scattering correction. A typical packed powder superleak might have 25% solid volume, in which case it would probably have a scattering correction of a little bit more than 1.1. The scattering correction is discussed further in Chapter 3.

If HeII is admitted into a porous solid whose pores are not small enough to lock the normalfluid then naturally first and second sound will be observable. Just as for fourth sound in a superleak, a scattering correction must be applied to observed sound speed data here to make it agree with theory. In theory (Ch. 4), the same scattering correction applies to first, second, and fourth sound in the same porous solid.