Physics 5B Lab Manual

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Introduction

PURPOSE

The laws of physics are based on experimental and observational facts. Laboratory work is therefore an important part of a course in general physics, helping you develop skill in fundamental scientific measurements and increasing your understanding of the physical concepts. It is profitable for you to experience the difficulties of making quantitative measurements in the real world and to learn how to record and process experimental data. For these reasons, successful completion of laboratory work is required of every student.

PREPARATION

Read the assigned experiment in the manual before coming to the laboratory. Since each experiment must be finished during the lab session, familiarity with the underlying theory and procedure will prove helpful in speeding up your work. Although you may leave when the required work is complete, there are often "additional credit" assignments at the end of each write-up. The most common reason for not finishing the additional credit portion is failure to read the manual before coming to lab. We dislike testing you, but if your TA suspects that you have not read the manual ahead of time, he or she may ask you a few simple questions about the experiment. If you cannot answer satisfactorily, you may lose mills (see below).

RESPONSIBILITY AND SAFETY

Laboratories are equipped at great expense. You must therefore exercise care in the use of equipment. Each experiment in the lab manual lists the apparatus required. At the beginning of each laboratory period check that you have everything and that it is in good condition. Thereafter, you are responsible for all damaged and missing articles. At the end of each period put your place in order and check the apparatus. By following this procedure you will relieve yourself of any blame for the misdeeds of other students, and you will aid the instructor materially in keeping the laboratory in order.

The laboratory benches are only for material necessary for work. Food, clothing, and other personal belongings not immediately needed should be placed elsewhere. A cluttered, messy laboratory bench invites accidents. Most accidents can be prevented by care and foresight. If an accident does occur, or if someone is injured, the accident should be reported immediately. Clean up any broken glass or spilled fluids.

FREEDOM

You are allowed some freedom in this laboratory to arrange your work according to your own taste. The only requirement is that you complete each experiment and report the results clearly in your lab manual. We have supplied detailed instructions to help you finish the experiments, especially the first few. However, if you know a better way of performing the lab (and in particular, a different way of arranging your calculations or graphing), feel free to improvise. Ask your TA if you are in doubt.

LAB GRADE

Each experiment is designed to be completed within the laboratory session. Your TA will check off your lab manual and computer screen at the end of the session. There are no reports to submit. The lab grade accounts for approximately 15% of your course total. Basically, 12 points (12%) are awarded for satisfactorily completing the assignments, filling in your lab manual, and/or displaying the computer screen with the completed work. Thus, we expect every student who attends all labs and follows instructions to receive these 12 points. If the TA finds your work on a particular experiment unsatisfactory or incomplete, he or she will inform you. You will then have the option of redoing the experiment or completing it to your TA's satisfaction. In general, if you work on the lab diligently during the allocated two hours, you will receive full credit even if you do not finish the experiment.

Another two points (2%) will be divided into tenths of a point, called "mills" (1 point = 10 mills). For most labs, you will have an opportunity to earn several mills by answering questions related to the experiment, displaying computer skills, reporting or printing results clearly in your lab manual, or performing some "additional credit" work. When you have earned 20 mills, two more points will be added to your lab grade. Please note that these 20 mills are additional credit, not "extra credit". Not all students may be able to finish the additional credit portion of the experiment.

The one final point (1%), divided into ten mills, will be awarded at the discretion of your TA. He or she may award you 0 to 10 mills at the end of the course for special ingenuity or truly superior work. We expect these "TA mills" to be given to only a few students in any section. (Occasionally, the "TA mills" are used by the course instructor to balance grading differences among TAs.)

If you miss an experiment without excuse, you will lose two of the 15 points. (See below for the policy on missing labs.) Be sure to check with your TA about making up the computer skills; you may be responsible for them in a later lab. Most of the first 12 points of your lab grade is based on work reported in your manual, which you must therefore bring to each session. Your TA may make surprise checks of your manual periodically during the quarter and award mills for complete, easy-to-read results. If you forget to bring your manual, then record the experimental data on separate sheets of paper, and copy them into the manual later. However, if the TA finds that your manual is incomplete, you will lose mills.

In summary:

Lab grade =	(12.0 points)
	− (2.0 points each for any missing labs)
	+ (up to 2.0 points earned in mills of "additional credit")
	+ (up to 1.0 point earned in "TA mills")
Maximum score =	15.0 points

Typically, most students receive a lab grade between 13.5 and 14.5 points, with the few poorest students (who attend every lab) getting grades in the 12s and the few best students getting grades in the high 14s or 15.0. There may be a couple of students who miss one or two labs without excuse and receive grades lower than 12.0.

How the lab score is used in determining a student's final course grade is at the discretion of the individual instructor. However, very roughly, for many instructors a lab score of 12.0 represents approximately B− work, and a score of 15.0 is A+ work, with 14.0 around the B+/A− borderline.

POLICY ON MISSING EXPERIMENTS

1. In the Physics 6 series, each experiment is worth two points (out of 15 maximum points). If you miss an experiment without excuse, you will lose these two points.

2. The equipment for each experiment is set up only during the assigned week; you cannot complete an experiment later in the quarter. You may make up no more than one experiment per quarter by attending another section during the same week and receiving permission from the TA of the substitute section. If the TA agrees to let you complete the experiment in that section, have him or her sign off your lab work at the end of the section and record your score. Show this signature/note to your own TA.

3. (At your option) If you miss a lab but subsequently obtain the data from a partner who performed the experiment, and if you complete your own analysis with that data, then you will receive one of the two points. This option may be used only once per quarter.

4. A written, verifiable medical, athletic, or religious excuse may be used for only one experiment per quarter. Your other lab scores will be averaged without penalty, but you will lose any mills that might have been earned for the missed lab.

5. If you miss three or more lab sessions during the quarter for any reason, your course grade will be Incomplete, and you will need to make up these experiments in another quarter. (Note that certain experiments occupy two sessions. If you miss any three sessions, you get an Incomplete.)

Fluids and Thermodynamics

APPARATUS

Shown in the pictures below:

- Electric tea kettle for hot water
- Temperature sensor
- Copper can with attached tube and quick-disconnect
- Rubber bands or tape to tape temperature sensor to metal can
- Low-pressure sensor

Not shown in the pictures above:

- Computer and ScienceWorkshop interface
- Tall container with tubing and quick disconnector for pressure-versus-depth measurements
- Meter stick
- 1000-ml beaker
- Rectangular aluminum and brass blocks
- Spring scale
- Acculab digital scale
- Vernier calipers
- 50-, 100-, and 200-g masses
- Barometer in room
- Paper towels and/or sponges in room to clean up splashed water

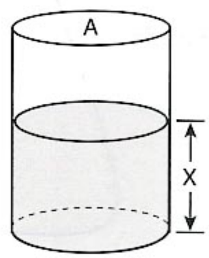
INTRODUCTION

In this experiment, you will measure the pressure of water as a function of depth, investigate the buoyant force exerted by water on submerged objects, and use ideal-gas-law properties of air to deduce and calculate the coldest possible temperature, called *absolute zero.* (More properly, one should say absolute zero is the lower limit of cold temperatures, since absolute zero can be approached but not reached.)

PASCAL'S LAW

Pressure, by definition, is force per unit area. In SI units, pressure is measured in Newtons per square meter, which is also called Pascals (Pa). Atmospheric pressure is approximately 100,000 Pa or 100 kilopascals (kPa). The pressure sensor used in this experiment reads in kPa.

To determine the variation of pressure with depth, consider a cylindrical slab of liquid of cross-sectional area A and height x.



(The slab need not be a circular cylinder; the cross-sectional area can have any shape as long as the sides are vertical.) We consider only the case in which the liquid is incompressible and has constant density ρ_l. The mass of liquid in the slab is equal to the density multiplied by the volume — $\rho_l Ax$ — while the weight of the liquid is $\rho_l gAx$. Thus, the pressure P exerted by the liquid on the bottom of the slab is equal to the weight divided by the area,

$$P = \rho_l g x, \tag{1}$$

and we see that the pressure is proportional to the depth x. This result is known as *Pascal's Law.*

ARCHIMEDES' PRINCIPLE

Archimedes' Principle tells us that an object completely or partially submerged in a fluid (liquid or gas) experiences an upward buoyant force. The buoyant force is equal in magnitude to the weight of the fluid displaced by the object. Suppose we place a rubber duck in a bathtub of water. The duck is less dense than the water, so it naturally floats. If we push down on the duck, though, we feel some "resistance" as the duck enters the water. This "resistance" is the upward buoyant force exerted by the water on the duck. We also observe that the duck sweeps aside — or displaces — a certain amount of water to make room for itself. When the duck is completely submerged in water, the displaced volume of water is equal to the total volume of the duck.

In general, to determine the magnitude of the buoyant force that a liquid exerts on a submerged object, we must consider how much of the object (of total volume V) lies below the liquid surface. Since only the submerged portion of the object displaces liquid, we see that the displaced volume of liquid is equal to the object's submerged volume. If we call the displaced volume V_d, then the mass of the displaced liquid is $\rho_\mathrm{l} V_\mathrm{d}$, and the weight of the displaced liquid is $\rho_\mathrm{l} g V_\mathrm{d}$. From Archimedes' Principle, the magnitude of the buoyant force B must be equal to this weight:

$$B = \rho_\mathrm{l} g V_\mathrm{d}. \tag{2}$$

For an object completely submerged in the liquid, $V_\mathrm{d} = V$. However, for an object only partially submerged, $V_\mathrm{d} < V$.

When an object suspended from a spring scale is lowered slowly into a liquid, the forces acting on the object are the (upward) spring force F_s, the (upward) buoyant force B, and the (downward) gravitational force or weight W. Since the object is essentially stationary, these three forces must balance:

$$\sum Fy = F_\mathrm{s} + B - W = 0. \tag{3}$$

Therefore, the reading of the spring scale is

$$F_\mathrm{s} = W - B. \tag{4}$$

This reading is known as the *apparent weight* of the object. Calling ρ_o the density of the object, we see that $W = mg = \rho_\mathrm{o} g V$. From this result and Eq. 2, it follows that

$$F_\mathrm{s} = \rho_\mathrm{o} g V - \rho_\mathrm{l} g V_\mathrm{d} = (\rho_\mathrm{o} V - \rho_\mathrm{l} V_\mathrm{d}) g. \tag{5}$$

THE IDEAL GAS LAW

Experimentally, it is found that any sufficiently rarefied gas satisfies the *ideal gas law*:

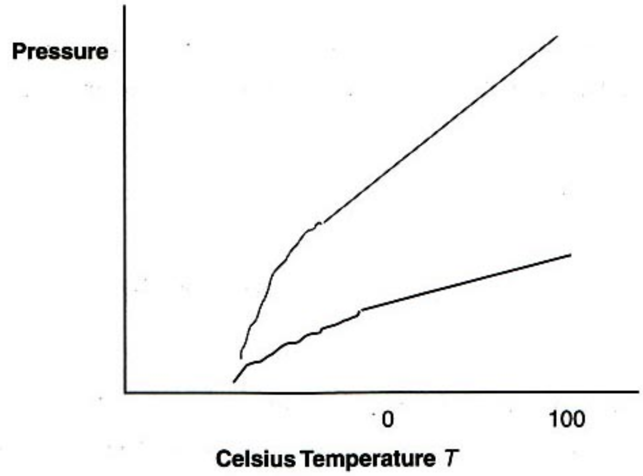
$$PV = nRT, \tag{6}$$

where P is the pressure, V is the volume, T is the absolute temperature, n is the number of moles of gas, and R is the universal gas constant (8.314 J/mol K). Historically, the different dependencies codified in this equation were named after the scientists who discovered them:

- Boyle's Law: P is inversely proportional to V (if T is held constant)
- Charles' Law: V is directly proportional to T (if P is held constant)
- Gay-Lussac's Law: P is directly proportional to T (if V is held constant)

MEASURING ABSOLUTE ZERO WITHOUT RISKING FROSTBITE

If we take different types of gases in various volumes — say, a liter of hydrogen, a cubic meter of helium, 500 cubic centimeters of air, a cubic centimeter of chlorine, etc. — and measure the dependencies of pressure P on Celsius temperature T at constant volume, we would obtain curves such as those shown below:



In each case, the pressure-versus-temperature relationship is linear at sufficiently high temperatures (i.e., the ideal-gas behavior); but as the temperature is reduced, each gas eventually deviates from the straight-line relationship. At sufficiently cold temperatures, the gas liquefies. However, the extrapolation of the linear part of the pressure-versus-temperature curve in each case intersects the temperature axis at the same point T_0 — the temperature we call *absolute zero* — irrespective of the type of gas or its initial pressure and volume.

Thus, we can take any gas (e.g., air), measure its pressure dependence on Celsius temperature at convenient values near room temperature, extrapolate the curve, and determine the value of absolute zero. For example, if we measure $P = mT + b$, where m is the slope of the P vs. T curve and b is the pressure at zero degrees Celsius, then the extrapolated intercept on the axis where $P = 0$ is the Celsius temperature of absolute zero:

$$T_0 = -b/m. \tag{7}$$

(If the chosen gas were very "non-ideal" at room temperature and atmospheric pressure, then we might need to reduce the pressure until its behavior approaches the ideal limit.) In this experiment, we will be using air at pressures near atmospheric and temperatures between the boiling and freezing points of water.

EQUIPMENT

In the first part of this experiment, pressure is measured by a small pressure sensor box which connects to one of the analog plugs of the Pasco interface. The sensor box sends a voltage proportional to the pressure toward the interface. The interface then converts the analog voltage to a digital signal and sends it to the computer. You can enter the pressure into tables, graph it, and so forth. The actual gas or liquid pressure is delivered to the pressure port of the sensor box by a plastic tube with a "quick-disconnector" piece which fits into the port.

Since you will now be dealing with water, be very careful not to get it on the computer, the keyboard, or the interface box. The tall container has two stopcocks; the lower one has a hose with a quick-disconnector on the end. Make sure both stopcocks are closed, and hook the quick-disconnector to the pressure sensor box. (The stopcocks are closed when their T-handles are turned perpendicular to the direction of liquid flow, and open when the T-handles are parallel to the liquid flow.) Place a plastic meter stick into the container so you can read the depth of water in centimeters.

INITIAL SETUP

1. Plug the cable from the pressure sensor box into analog channel A of the signal interface.

2. Turn on the signal interface and the computer.

3. Call up Capstone and choose the "Graph & Digits" option. In the "Hardware Setup" tab, click on channel A and select "Pressure Sensor, Low". A pressure sensor symbol appears under analog channel A.

4. In the digits box, click "Select Measurement" and choose "Pressure (kPa)".

5. Add some water to the tall container, and place the beaker to catch water from the upper stopcock. Use a small piece of hose from the upper stopcock to the beaker. The pressure of the full column will shoot water over the beaker if the upper stopcock is fully opened. The hose from the closed lower stopcock should be connected to the pressure sensor box.

6. Click the "Record" button, and check that you are obtaining gauge pressure readings. (The gauge pressure is the pressure exerted by the water, and is equal to the difference between the absolute and atmospheric pressures. Equivalently, it is equal to the pressure above atmospheric.) When you open the lower stopcock, the pressure increases. Although the trapped air in the tube should prevent any water from getting into the sensor, take caution not to disconnect the quick-disconnector from the pressure sensor before turning off the stopcock. Allow some water to drain out of the tall container into the beaker while observing the pressure reading. The reading should, of course, decrease as the water drains out. Stop recording and click "Delete Last Run" to discard this data set.

PROCEDURE PART 1: PRESSURE

1. Click on the table icon on the right side of the screen and drag a new table to your work space. Click "Select Measurement" on the first column of the table and choose "Pressure (kPa)". Click "Select Measurement" on the second column. In the "Create New" option, choose "User-Entered Data". You will be entering the water depth in this column. Feel free to Change the title of this column.

2. On the bottom of the screen, click "Continuous Mode" and change this to "Keep Mode".

3. Pour water into the container until it is about 75 cm deep. Note that the stopcock to the pressure sensor is 3 cm from the bottom, so you need to subtract 3 cm from all of the meterstick-level readings. Be sure to convert the level readings to meters before typing them in. Open the stopcock to the pressure sensor.

4. Click "Preview" to initiate the sensor. Then click "Keep Sample" to have the pressure reading added to the table. Enter the depth reading in meters (3 cm less than the meterstick reading)

in the second column of your table. Take pressure readings at each of the following marks on the meter stick: 75, 70, 65, 60, 55, 50, 45, 40, 35, 30, 25, and 20 cm.

5. For the final entry, type in zero for the depth. Close the lower stopcock, disconnect the hose from the sensor, and click "Keep Sample" to record the gauge pressure of the atmosphere (which should be zero). Again, the stopcocks are open when their T-shaped tops are aligned with their tubes, and closed otherwise. Click the "Stop" button to end the data run.

6. Click the "Select Measurement" botton on the y-axis and choose "Pressure (kPa)" and put your depth recordings on the y-axis. Click the "Apply Selected curve fits..." tool and select "Linear". A box should appear that tells you the slope and y-intercept of the best-fit line. According to the equation of pressure versus depth ($P = \rho_l g x$), the slope of the graph should be $\rho_l g$. Since the density of water is 1000 kg/m^3, $\rho_l g$ = 9800 N/m^3. However, the pressure readings were taken in kPa, so we expect the slope to be 9.8 kN/m^3 (kN = kilonewtons). Record the experimental value of the slope in the "Data" section, and compare it with the theoretical value.

PROCEDURE PART 2: BUOYANCY

In this part, you will determine whether the buoyant force exerted by water on a submerged block depends on the density of the block.

1. Using the Vernier calipers, measure the dimensions of the aluminum block, and calculate its total volume.

2. Pour water into the beaker until it is almost full. Attach the aluminum block to the spring scale with its longest dimension vertical, and lower the block <u>very slowly</u> into the beaker. Record the reading of the spring scale when the block is 1/5, 2/5, 3/5, 4/5, and fully submerged in water, using the 1-cm marks along the block as a guide, and convert these readings into weight (in SI units of Newtons). Determine the displaced volume of water for each case.

3. Using the known densities of aluminum (2700 kg/m^3) and water (1000 kg/m^3), plot the apparent weight F_s of the block as a function of the displaced volume of water V_d. (Recall that the apparent weight is equal to the reading of the spring scale converted into SI units of newtons.) Make a best-fit line through your data points, and calculate the slope of this line. How well does your graph obey a linear fit? Determine the theoretical slope of the line from Eq. 5, and compare it with your experimental value.

4. Repeat steps 1 – 3 with the brass block (of density 8400 kg/m^3).

5. Based on your results above, comment on whether the buoyant force depends on the density of the block.

PROCEDURE PART 3: IDEAL GAS AND ABSOLUTE ZERO

1. <u>Safety Considerations</u>: When the experimental steps below call for "hot water", use hot tap water or water heated by the electric tea kettles to 70 – 80 °C: hot to the touch, <u>but not scalding hot</u>. Use great care not to spill or splash water on the keyboard or other computer

equipment. Clean up any splashed or spilled water immediately with a sponge and/or paper towels.

2. Determining Absolute Zero: You will be producing a graph of pressure as a function of temperature, then extrapolating the graph to zero pressure to determine the value of absolute zero.

   a. Fill the electric tea kettle with water from the faucet in the lab room. Connect the low-pressure sensor to the tube from the cylindrical copper can, fix the thermometer sensor to the can using rubber bands or tape, and submerge the can in the water inside the kettle.

   b. Get a new Capstone page. Set up the "Temperature Sensor" and "Pressure Sensor, Low" in channels A and B of the interface, and have them take data once per second (change the sample rate at the bottom of the screen to 1 Hz). Turn on the tea kettle and start recording data. Record until the pressure changes by about 8 kPa, then stop recording and turn off the kettle. (The data should look linear.) While recording you can perform the next steps below.

   c. During or after recording your data, you can set up the calculation for your graph. You will plot a graph of temperature versus pressure. The pressure sensor measures deviations from the ambient atmospheric pressure in kPa. But since you need the total pressure, you will have to add the ambient atmospheric pressure to your reading. Read the barometer in the room, and convert the reading to kPa. The conversions below may help:

$$1 \text{ bar} = 10^5 \text{ Pa} = 100 \text{ kPa}. \tag{8}$$

$$1 \text{ atm} = 1.013 \times 10^5 \text{ Pa} = 101.3 \text{ kPa} = 0.760 \text{ m of mercury}. \tag{9}$$

   d. Select your data using the "Highlight range of points..." tool. Click on the "Apply selected curve fits..." tool and choose "Linear". A box should appear telling you the slope and y-intercept of the best-fit line.

   e. Add the ambient atmospheric pressure to the y-intercept value of your best-fit line. Use the slope obtained from the best-fit line and this new y-intercept to calculate the estimate of absolute zero, $T_0 = -b/m$. Calculate your experimental error using the known value of absolute zero.

DATA

Procedure Part 1:

3. Pressure at depth of 0.72 m = ______________________________

   Pressure at depth of 0.67 m = ______________________________

   Pressure at depth of 0.62 m = ______________________________

   Pressure at depth of 0.57 m = ______________________________

Pressure at depth of 0.52 m = ______________________________

Pressure at depth of 0.47 m = ______________________________

Pressure at depth of 0.42 m = ______________________________

Pressure at depth of 0.37 m = ______________________________

Pressure at depth of 0.32 m = ______________________________

Pressure at depth of 0.27 m = ______________________________

Pressure at depth of 0.22 m = ______________________________

Pressure at depth of 0.17 m = ______________________________

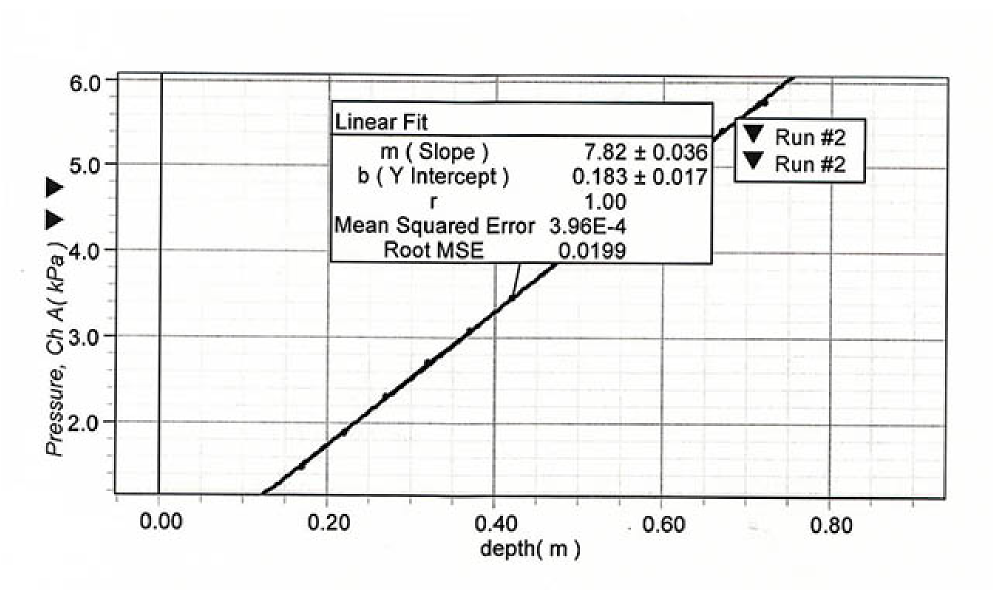
4. Pressure at depth of 0.00 m = ______________________________

5. Slope of line (experimental) = ______________________________

   Slope of line (theoretical) = ______________________________

   Percentage difference in slope of line = ______________________________

   You may print the data table and graph showing pressure as a function of depth.



Procedure Part 2:

1. Dimensions of aluminum block = ______________________________

   Volume of aluminum block = ______________________________

2. Reading of spring scale when aluminum block is 1/5 submerged = ________________________

Reading of spring scale when aluminum block is 2/5 submerged = ________________________

Reading of spring scale when aluminum block is 3/5 submerged = ________________________

Reading of spring scale when aluminum block is 4/5 submerged = ________________________

Reading of spring scale when aluminum block is fully submerged = ________________________

Displaced volume of water when aluminum block is 1/5 submerged = ______________________

Displaced volume of water when aluminum block is 2/5 submerged = ______________________

Displaced volume of water when aluminum block is 3/5 submerged = ______________________

Displaced volume of water when aluminum block is 4/5 submerged = ______________________

Displaced volume of water when aluminum block is fully submerged = _____________________

3. Slope of best-fit line = ______________________________

   Theoretical slope of line = ______________________________

   Percentage difference in slope of line = ______________________________

4. Dimensions of brass block = ______________________________

   Volume of brass block = ______________________________

   Reading of spring scale when brass block is 1/5 submerged = ___________________________

   Reading of spring scale when brass block is 2/5 submerged = ___________________________

   Reading of spring scale when brass block is 3/5 submerged = ___________________________

   Reading of spring scale when brass block is 4/5 submerged = ___________________________

   Reading of spring scale when brass block is fully submerged = ___________________________

   Displaced volume of water when brass block is 1/5 submerged = _________________________

   Displaced volume of water when brass block is 2/5 submerged = _________________________

   Displaced volume of water when brass block is 3/5 submerged = _________________________

   Displaced volume of water when brass block is 4/5 submerged = _________________________

   Displaced volume of water when brass block is fully submerged = _________________________

   Plot the graph of F_s as a function of V_d using one sheet of graph paper at the end of this workbook. Remember to label the axes and title the graph.

   Slope of best-fit line= ______________________________

   Theoretical slope of line = ______________________________

Percentage difference in slope of line = ______________________________

5. Does the buoyant force depend on the density of the block?

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Driven Harmonic Oscillator

APPARATUS

- Computer and interface
- Mechanical vibrator and spring holder
- Stands, etc. to hold vibrator
- Motion sensor
- C-209 spring
- Weight holder and five 100-g mass disks

INTRODUCTION

This is an experiment in which you will plot the resonance curve of a driven harmonic oscillator. Harmonic oscillation was covered in Physics 6A, so we include a partial review of both the underlying physics and the Pasco Capstone. We will continue to give fairly detailed instructions for taking data in this first Physics 6B lab. However, this is the last experiment with detailed instructions on setting up Pasco Capstone. Henceforth, it will be assumed that you know how to connect the cables to the computer and to the sensors; how to call up a particular sensor; and how to set up a table, graph, or digit window for data taking.

THEORY

Hooke's Law for a mass attached to a spring states that $F = -kx$, where x is the displacement of the mass from equilibrium, F is the restoring force exerted by the spring on the mass, and k is the (positive) spring constant. If this force causes the mass m to accelerate, then the equation of motion for the mass is

$$-kx = ma. \tag{1}$$

Substituting for the acceleration $a = d^2x/dt^2$, we can rewrite Eq. 1 as

$$-kx = m\, d^2x/dt^2 \tag{2}$$

or

$$d^2x/dt^2 + \omega_0^2 x = 0, \tag{3}$$

where $\omega_0 = \sqrt{k/m}$ is called the *resonance angular frequency* of oscillation. Eq. 3 is the differential equation for a *simple harmonic oscillator* with no friction. Its solution includes the sine and cosine functions, since the second derivatives of these functions are proportional to the negatives of the functions. Thus, the solution $x(t) = A\sin(\omega_0 t) + B\cos(\omega_0 t)$ satisfies Eq. 3. The parameters A and B are two constants which can be determined by the initial conditions of the motion. The natural frequency f_0 of such an oscillator is

$$f_0 = \omega_0/2\pi = (1/2\pi)\sqrt{k/m}. \tag{4}$$

In the simple case described above, the oscillations continue indefinitely. We know, however, that the oscillations of a real mass on a spring eventually decay because of friction. Such behavior is called *damped harmonic motion*. To describe it mathematically, we assume that the frictional force is proportional to the velocity of the mass (which is approximately true with air friction, for example) and add a damping term, $-b\,dx/dt$, to the left side of Eq. 2. Our equation for the damped harmonic oscillator becomes

$$-kx - b\,dx/dt = m\,d^2x/dt^2 \tag{5}$$

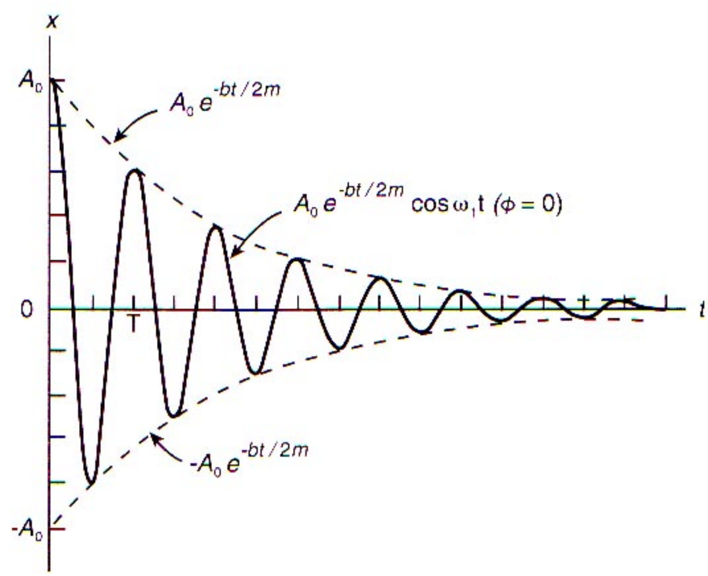
or

$$m\,d^2x/dt^2 + b\,dx/dt + kx = 0. \tag{6}$$

Physics texts give us the solution of Eq. 6 (and explain how it is obtained):

$$x(t) = A_0 e^{-bt/2m}\cos(\omega_1 t + \phi). \tag{7}$$

The parameter A_0 is the initial amplitude of the oscillations, and ϕ is the phase angle; these two constants are determined by the initial conditions of the motion. The oscillations decay exponentially in time, as shown in the figure below:



In addition, the angular frequency of oscillation is shifted slightly to

$$\omega_1 = \sqrt{k/m - (b/2m)^2} = \sqrt{\omega_0^2 - (b/2m)^2}. \tag{8}$$

Now imagine that an external force which varies cosinusoidally (or sinusoidally) in time is applied to the mass at an arbitrary angular frequency ω_2. The resultant behavior of the mass is known as *driven harmonic motion*. The mass vibrates with a relatively small amplitude, unless the driving angular frequency ω_2 is near the resonance angular frequency ω_0. In this case, the amplitude becomes very large. If the external force has the form $F_\mathrm{m}\cos(\omega_2 t)$, then our equation for the driven harmonic oscillator can be written as

$$-kx - b\,dx/dt + F_\mathrm{m}\cos(\omega_2 t) = m\,d^2x/dt^2 \tag{9}$$

or

$$m\,d^2x/dt^2 + b\,dx/dt + kx = F_{\mathrm{m}}\cos(\omega_2 t). \tag{10}$$

The solution of Eq. 10 can also be found in physics texts:

$$x(t) = (F_{\mathrm{m}}/G)\cos(\omega_2 t + \phi), \tag{11}$$

where

$$G = \sqrt{m^2(\omega_2^2 - \omega_0^2)^2 + b^2\omega_2^2} \tag{12}$$

and

$$\phi = \cos^{-1}(b\omega_2/G). \tag{13}$$

The factor G in the denominator of Eq. 11 determines the shape of the resonance curve, which we wish to measure in this experiment. When the driving angular frequency ω_2 is close to the resonance angular frequency ω_0, G is small, and the amplitude of oscillation becomes large. When the driving angular frequency ω_2 is far from the resonance angular frequency ω_0, G is large, and the amplitude of oscillation is small.

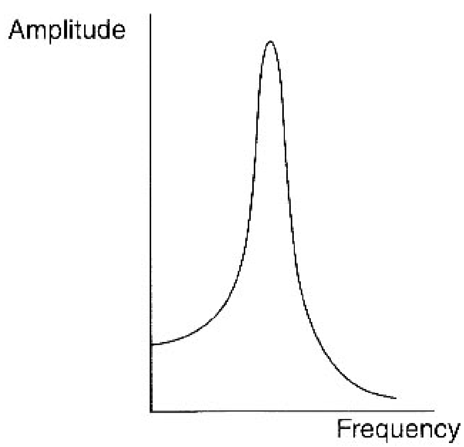


Figure 2

This is the curve we wish to measure in the experiment.

THE QUALITY FACTOR

The sharpness of the resonance curve is determined by the quality factor (or Q value), Q. If the frictional force, measured by the parameter b, is small, then Q is large, and the resonance curve is sharply peaked. If the frictional force is large, then Q is small, and the resonance curve is broad (Figure 3).

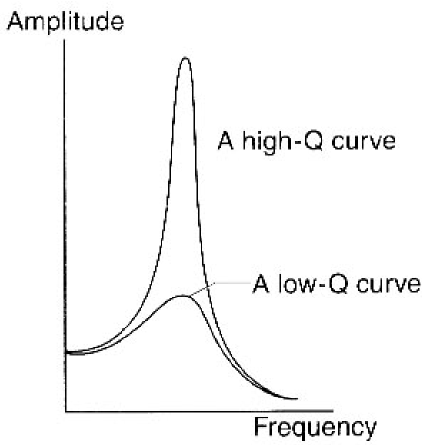


Figure 3

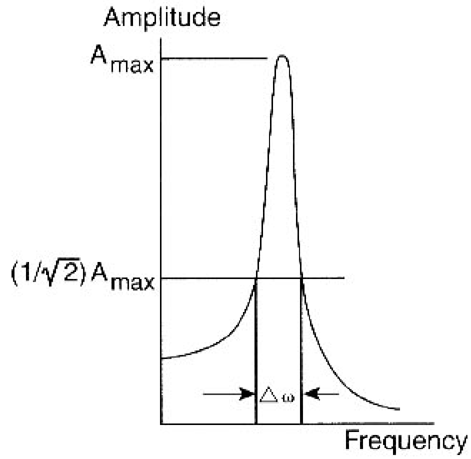


Figure 4

The general definition of Q is

$$Q = 2\pi \times \text{(energy stored) / (energy dissipated in one cycle).} \tag{14}$$

(If the energy dissipated per cycle is small, then Q is large, and the resonance curve is sharply peaked.) Physics texts derive the relationship between Q and the motion parameters,

$$Q = m\omega_1/b, \tag{15}$$

and relate Q to the sharpness of the resonance peak,

$$Q = \omega_1/\Delta\omega, \tag{16}$$

where $\Delta\omega = \omega_{\text{high}} - \omega_{\text{low}}$ is the difference in angular frequencies at which the amplitude has dropped to $1/\sqrt{2}$ of its maximum value (see Figure 4). Note that Q also controls the exponential damping factor, $e^{-bt/2m}$, in Eq. 7 and Figure 1. Using Eq. 15, we can show that

$$e^{-bt/2m} = e^{-\omega_1 t/2Q}. \tag{17}$$

The reciprocal of the factor which multiplies t in the exponent, $2m/b = 2Q/\omega_1$, is the time required for the amplitude of oscillation to decay to $1/e$ of its initial value (the so-called "e-folding time").

EXPERIMENTAL SETUP

This experiment utilizes a signal interface which drives a mechanical vibrator attached to a spring with a mass. We will measure the position of the mass by echo location using the motion sensor (sonic ranger).

1. Plug the yellow-banded cable of the motion sensor into digital channel 1 of the signal interface and the other cable into digital channel 2. From the mechanical vibrator, plug the red and black wires into the output of the signal interface.

2. Turn on the signal interface and the computer.

3. Call up PASCO Capstone. Click on "Hardware Setup" to display the interface. Click on channel 1 of the interface and select "Motion Sensor II". Click on the yellow circle at the output of the interface. This will add the Output Voltage-Current Sensor.

4. Click on "Signal Generator". Set Waveform to Sine, set Frequency to 10 Hz, and set Amplitude to 1 V.

5. Click the "On" switch in the signal generator window, and check that the mechanical vibrator stem is shaking. Experiment with the up and down arrows to adjust the frequency and amplitude of the vibrations. You can also click on the number itself and type the desired value. Then click "Off".

PROCEDURE PART 1: FINDING THE NATURAL FREQUENCY

1. Attach a C-209 spring with a total mass of 450 g (400-g mass + 50-g mass holder) to the vibrator stem. Place the motion sensor on the floor under the mass-spring system.

2. <u>In the following procedures, be very careful not to drop masses onto the motion sensor. Secure the spring holder firmly to the vibrator stem. If the vibrations become large, they might shake a mass loose. Do not leave masses unattended on the spring; set them aside immediately when you stop taking measurements for a while.</u>

3. Select the "Text & Graph" option on Capstone. Click on the "Select Measurement" button on the y-axis of the graph. Under Motion Sensor II, click on "position".

4. Make sure the signal generator switch is "Off" in its window. With your hand, set the mass gently vibrating, click "Record", then "Stop" after approximately 15 seconds.

5. To zoom in on your data, click on the "Scale axes to show all data" button above the graph (square with a red arrow pointing diagonally).

6. Check that you are obtaining clear oscillations on the graph. If not, adjust the positions of the motion sensor and spring vibrator accordingly. You can delete experimental runs by clicking the drop down arrow next to "Delete Last Run". Record a clear set of 12 – 15 oscillations.

7. Note a set of 10 clear oscillations on your graph. Record the time at the beginning of the oscillations and the time at the end of 10 complete oscillations. The frequency (number of oscillations per second) is equal to 10 divided by the time required for 10 complete oscillations.

8. Repeat the procedure above three times, and record the average frequency. (This is sometimes called the natural frequency.)

PROCEDURE PART 2: PLOTTING THE RESONANCE CURVE

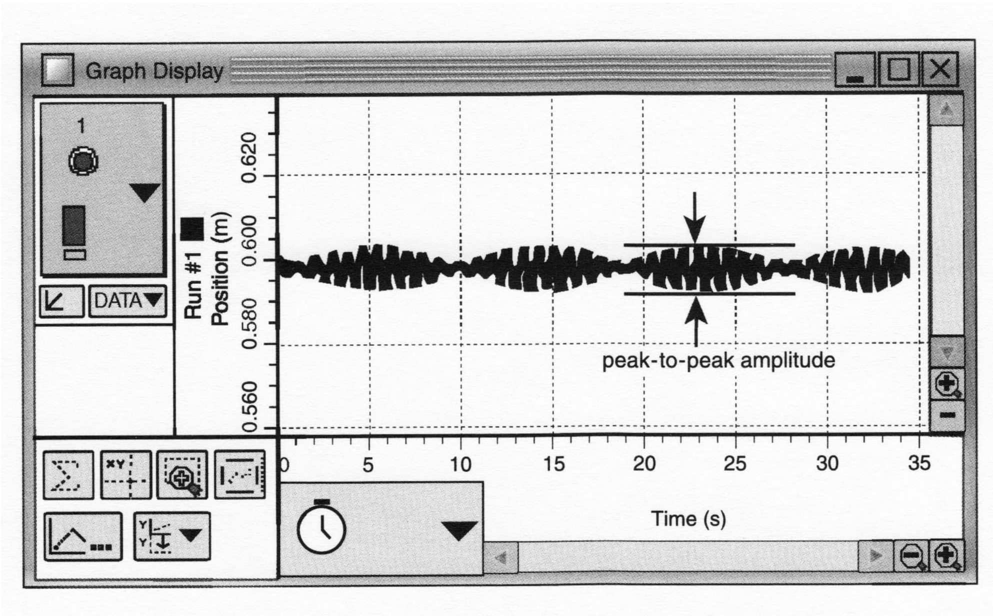
In this section, you will verify that resonance occurs when a driving force is applied at the natural frequency of the oscillator.

1. Set the frequency of the signal generator to your measured natural frequency and the amplitude to approximately 1 V. Click the "Auto" box on the signal generator window. This will

automatically turn the generator on when the "Record" button is clicked and switch it off when "Stop" is clicked.

2. Set the mass at rest, and click "Start". Observe the oscillations building up. Make sure they do not get too wild; if so, stop and reduce the amplitude of the signal generator, and start again.

3. Take all your measurements below at the same driving amplitude. First, locate the resonance, set a reasonable driving amplitude, and proceed with measurements on either side of the resonance. (You may find that the oscillations build up and later decay. This is fine. When you are not exactly on resonance, the driven oscillations are "beating" against the natural frequency. At resonance, the amplitude builds up to a large value and remains there. The pattern above is a "transient" which will decay eventually (at least in the ideal case).)



4. Change the frequency by small steps until you locate the resonance exactly, and record the amplitude of oscillation with the frequency in the "Data" section.

5. When taking your measurements, expand the y-scale by clicking and dragging on the y-axis, read off the extremes of the oscillations at their maximum amplitude (as in the illustration above), and record the values in the "Data" section. The difference of the two extreme numbers is, of course, the peak-to-peak amplitude of oscillation. Compute this amplitude, and record it in the "Data" section. Be sure your mass is exactly at rest before starting a run.

6. Take measurements at different frequencies until you have a series of measurements (say, 10 or more) which cross the resonance, and continue on either side far enough so the oscillations are quite small compared to the maximum at resonance. Use small frequency steps with the up and down arrows near the resonance to map it accurately and obtain a nice smooth curve. You can adjust the number of decimal places by clicking on the left and right arrows near the frequency setting.

7. When you have a good series of measurements across the resonance, make a careful plot of

the resonance curve in Excel. Always title your graph and label the axes. Show your work to the TA. You have completed the required part of the experiment. The next steps result in varying degrees of additional credit.

DATA

Procedure Part 1:

8. Frequency (Trial 1) = ______________________________

   Frequency (Trial 2) = ______________________________

   Frequency (Trial 3) = ______________________________

9. Average frequency = ______________________________

Procedure Part 2:

5. Amplitude = ______________________________

   Frequency = ______________________________

6. Maximum position = ____________________________________

   Minimum position = ______________________________

   Peak-to-peak amplitude = ______________________________

7. Maximum positions = ____________________________________

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   Minimum positions = ______________________________

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Peak-to-peak amplitudes = ______________________________

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8. Plot the resonance curve using Excel. Remember to label the axes and title the graph.

ADDITIONAL CREDIT PART 1: FINDING THE SPRING CONSTANT (2 mills)

We determined the spring constant k for springs several times in the Physics 6A lab by using a meter stick and a set of masses, entering the measurements into Excel, and finding the slope of the line.

Here is an alternate method. Use the motion sensor to take the position measurements. Get rid of any previous data runs by clicking the drop down arrow under “Delete Last Run” and clicking “Delete All Runs”. Drag a table symbol over to your work space. Click the “Select Measurement” button and choose “Position”. Make sure that the Signal Generator is in the “Off” setting and not “Auto”.

Change the recording mode from “Continuous Mode” to “Keep Mode” at the bottom of the screen.

Start with only one of the five masses on the hanger. With the spring system still directly above the motion sensor, click "Preview". With the mass at rest, click the "Keep Sample" button. Add another mass and click the "Keep Sample" button again. Do this until you have values recorded for 5 different masses. Click the "Stop" button.

Use these 5 values of position and the five masses you used to make a plot of force vs displacement. Use Excel to do a best-fit line to your data and determine the slope. How is the slope of your graph related to the spring constant? You will need to use the correct units of force (Newtons) to get k in the proper units. You could either convert each mass entry into units of force and make your keyboard entries in Newtons, or just proceed with mass in units of kilograms and make the conversion of units at the end when you read the slope from the curve-fitting routine. In your work, make clear which method you are using.

Slope of line = ____

Spring constant k = ____

ADDITIONAL CREDIT PART 2: PREDICTING THE RESONANCE FREQUENCY (1 mill)

If there were no friction, the resonance frequency would be

$$f_0 = \omega_0/2\pi = (1/2\pi)\sqrt{k/m}. \tag{4}$$

Compute this value of f_0, and compare it with your measured value of the resonance frequency. Record your results clearly.

f_0 (computed from Eq. 4) = ____

f_0 (measured) = ____

Percentage difference in f_0 = ____

ADDITIONAL CREDIT PART 3: THE QUALITY FACTOR AND FRICTION (2 mills)

We saw that

$$Q = \omega_1/\Delta\omega, \tag{16}$$

where $\Delta\omega = \omega_{\text{high}} - \omega_{\text{low}}$. Compute the value of Q for your mass on a spring by finding the angular frequencies on your resonance curve where the amplitude has dropped to $1/\sqrt{2}$ of its maximum value.

Q is also equal to $m\omega_1/b$, by Eq. 15. By approximating ω_1 as ω_0, estimate the value of b. Then compute the true resonance angular frequency, including friction, from

$$\omega_1 = \sqrt{k/m - (b/2m)^2}. \tag{8}$$

What is the percentage difference between ω_1 and ω_0? Could your measurements of the resonance curve have distinguished between ω_1 and ω_0?

ω_{high} = ____

ω_{low} = ____

Q = ____

b = ____

ω_1 = ____

Percentage difference between ω_1 and ω_0 = ____________________________

Standing Waves

APPARATUS

- Computer and interface
- Mechanical vibrator
- Clip from vibrator to string
- Three strings of different densities
- Meter stick clamped vertically to measure vibration amplitudes
- Weight set
- Acculab digital scale

INTRODUCTION

Have you ever wondered why pressing different positions on your guitar string produces different pitches or sounds? Or why the same sound is produced by pressing certain positions on two or more strings? By exploring several basic properties of standing waves, you will be able to answer some of these questions. In this experiment, you will study standing waves on a string and discover how different modes of vibration depend on the frequency, as well as how the wave speed depends on the tension in the string.

THEORY: FORMATION OF STANDING WAVES

Consider a string under a tension F with its ends separated by a distance L. Figure 1 depicts a complex wave on the string, which could be produced by plucking the string or drawing a bow across it.

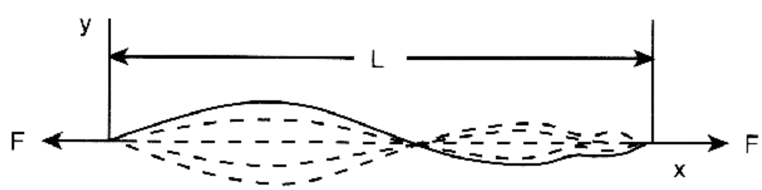


Figure 1

We will see that a complex wave such as this can be constructed from a sum of sinusoidal waves. Therefore, this focus of this experiment is on sinusoidal waves.

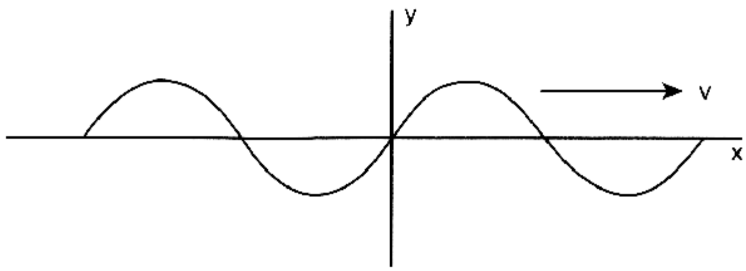


Figure 2

Figure 2 shows a wave traveling along the x-axis. The equation describing the motion of this wave is based on two observations. First, the shape of the wave does not change with time (t). Second, the position of the wave is determined by its speed in the x direction. Based on these observations, we see that the vertical displacement of the wave (y) is a function of both x and t. Let $y(x, t = 0) = f(x)$, where $f(x)$ represents the function that characterizes the shape of the wave. Then $y(x, t) = f(x - vt)$, where v is the speed of the wave. Although this description holds true for all traveling waves, we will limit our discussion to sinusoidal waves.

The vertical displacement of the traveling sinusoidal wave shown in Figure 2 can be expressed as

$$y(x, t) = A \sin[(2\pi/\lambda)(x - vt)], \tag{1}$$

where A is the *amplitude* of the wave (i.e., its maximum displacement from equilibrium) and λ is the *wavelength* (i.e., the distance between two points on the wave which behave identically). Expanding the term inside the brackets gives

$$y(x, t) = A \sin(2\pi x/\lambda - 2\pi vt/\lambda). \tag{2}$$

By substituting $\lambda = vT$, $k = 2\pi/\lambda$, and $\omega = 2\pi/T$ (where T is the *period*, k is the *angular wavenumber*, and ω is the *angular frequency*), we obtain

$$y(x, t) = A \sin(kx - \omega t). \tag{3}$$

As t increases, the argument of the sine function ($kx - \omega t$) decreases. In order to obtain the same value of y at a later time, x must also increase, which implies that this wave travels to the right. Conversely, the argument ($kx + \omega t$) represents a wave traveling to the left. When the right-traveling wave of Figure 2 reaches a fixed end of the string, it will be reflected in the opposite direction.

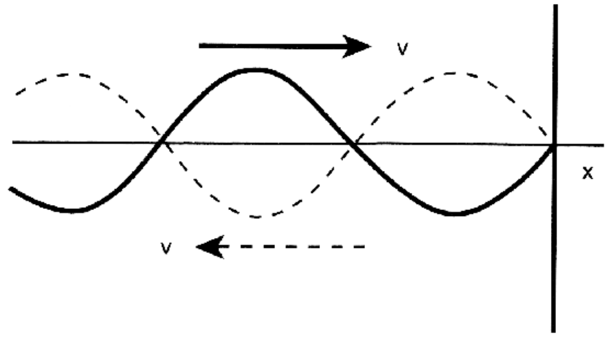


Figure 3

The right-moving incident wave, y_1, generates a left-moving reflected wave, y_2, with the same amplitude:

$$y_1(x,t) = A\sin(kx - \omega t) \tag{4}$$

$$y_2(x,t) = A\sin(kx + \omega t). \tag{5}$$

The resultant wave, y_3, which is the sum of the individual waves, is given by

$$y_3(x,t) = y_1(x,t) + y_2(x,t) = A\sin(kx - \omega t) + A\sin(kx + \omega t). \tag{6}$$

We can rewrite Eq. 6 by using the trigonometric identity:

$$A\sin(\alpha) + A\sin(\beta) = 2A\sin[(\alpha+\beta)/2]\,\cos[(\alpha-\beta)/2] \tag{7}$$

$$y_3(x,t) = 2A\sin(kx)\,\cos(\omega t). \tag{8}$$

Note that the x and t terms are separated such that the resultant wave is no longer traveling. Eq. 8 shows that all particles of the wave undergo simple harmonic motion in the y direction with angular frequency ω, although the maximum amplitude for a given value of x is bounded by $\mid 2A\sin(kx) \mid$. If we fix the two ends of the string and adjust the frequency so that an integral number of half waves fit into its length, then this *standing wave* is said to be in *resonance.*

The fixed ends impose a *boundary condition* on the string; its amplitude at the ends must be zero at all times. Thus, we can say that at $x = 0$ and $x = L$ (where L is the length of the string),

$$y_3(x=0,t) = y_3(x=L,t) = 0 \tag{9}$$

$$2A\sin(k\cdot 0)\,\cos(\omega t) = 2A\sin(kL)\,\cos(\omega t) = 0 \tag{10}$$

or

$$\sin(kL) = 0. \tag{11}$$

Eq. 11 is a boundary condition which restricts the string to certain modes of vibration. This equation is satisfied only when $kL = n\pi$, where n is the *index* of vibration and is equal to any positive integer. In other words, the possible values of k and λ for any given L are

$$kL = (2\pi/\lambda)L = n\pi \qquad (n = 1, 2, 3, \ldots) \tag{12}$$

or

$$\lambda = 2L/n. \tag{13}$$

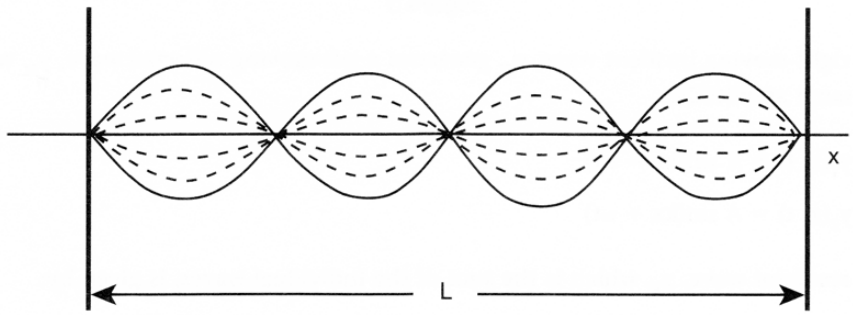


Figure 4

Figure 4 shows the mode with index $n = 4$: the fourth *harmonic*. The positions at which the vibration is small or zero are called *nodes*, while the positions where the vibration is largest are called *antinodes*. The number of antinodes is equal to the index of vibration and to the ordinal rank of the harmonic (fourth, in the case above). Figure 5 shows several other modes of vibration.

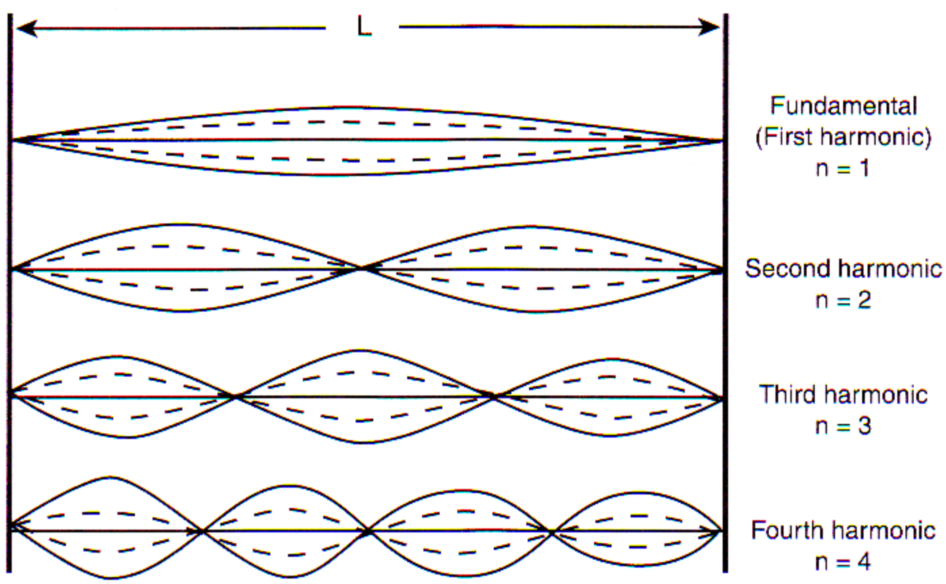


Figure 5

Note that wavelength $= 2 \times$(string length between supports)$/n$, where $n =$ mode number (or index) $=$ number of antinodes.

THEORY: PROPERTIES OF STANDING WAVES

The wave speed (v) depends on two quantities — frequency (f) and wavelength (λ) — which are related by

$$v = f\lambda. \tag{14}$$

A wave is created by exciting a stretched string; therefore, the speed of the wave also depends on the tension (F) and the mass per unit length of the string (μ). Physics texts give us the derivation of the wave speed:

$$v = (F/\mu)^{1/2}. \tag{15}$$

By combining Eqs. 13, 14, and 15, we can express the frequency as

$$f = v/\lambda = (n/2L)v = (n/2L)(F/\mu)^{1/2}. \tag{16}$$

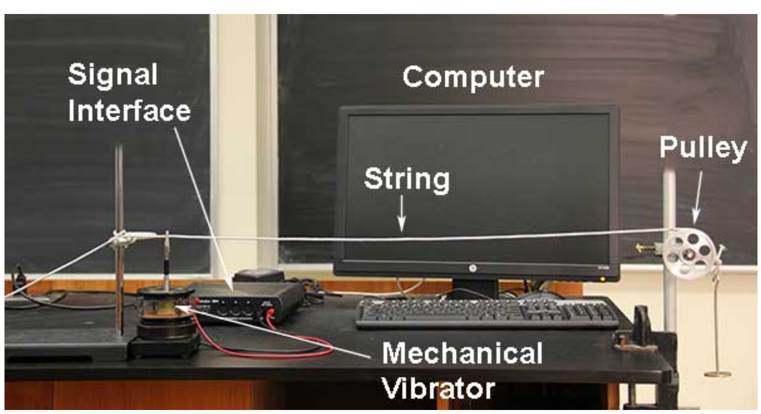
When a wave travels from one medium to another, some of its properties change (e.g., speed and wavelength), but its frequency remains fixed. For example, consider the point where two strings of different densities are joined. If the two strings have different frequencies, then the two sides of the point would oscillate at their own frequencies, and the point would no longer be a "joint". Mathematically speaking, the function describing the string would not be continuous at the "joint". The constancy of the frequency allows us to determine the wave speed and wavelength in a different medium, if the frequency in that medium is known.

EXPERIMENTAL SETUP

Set up the equipment as shown in Figure 6. Adjust the vibrator clamp on the side to position it firmly in the vertical orientation. Run one end of the string from a vertical bar past the vibrator and over the pulley. The vibrator is connected to the string with an alligator clip. Attach a mass hanger to the other end of the string. Throughout this experiment, you will be changing both the density of the medium (by using different strings) and the tension (by using different weights). To measure vibration amplitudes, it is helpful to have a meter stick clamped vertically near the string.

1. Turn on the signal interface and the computer.

2. Call up Capstone. Under "Hardware Setup", click on the output channels of the interface to connect the mechanical vibrator. Under "Signal Generator", click on "SW750 Output".



3. Note that a sine wave has already been selected. We will use only sine waves in this experiment. Set the amplitude and frequency of the signal generator initially to approximately +2 V and 20 Hz, respectively. Then click "On". You should see the string vibrate. Adjust the frequency to observe the multiple harmonics of the standing wave. Remember that you can obtain frequency steps of various sizes by clicking on the up and down arrows. Click the right and left arrows to adjust the size of these steps.

PROCEDURE: PART 1

In this section, we will keep the tension and density of the string constant to find experimentally the relationship between frequency and number of antinodes.

1. Adjust the frequency until you obtain a nice standing wave with two antinodes ($n = 2$). Record this frequency in the "Data" section.

2. Obtain and record the frequencies for consecutive n values. Take at least six measurements, starting with the fundamental mode.

3. Calculate and record the wavelength, Eq. 13, and wave speed, Eq. 14, corresponding to each n.

4. Plot a graph of frequency as a function of n. What is the relationship between the two variables?

PROCEDURE: PART 2

In this section, we will keep n constant and change the weights to find the relationship between frequency and tension.

1. Choose one of the three strings. If you like to see data that agrees well with theory, choose the finest string. If you would rather see more interesting data, for which you might need to explain the discrepancy, choose the most massive string. Measure and record the linear mass density ($\mu = M/L$) of the string by obtaining its total mass M and total length L. Use the digital scale to weigh the string. Keep all units in the SI system (kilograms and meters).

2. Using the 50-g mass hanger, measure and record the frequency for the $n = 2$ mode. (Note: You may choose any integer for n, but remember to keep n constant throughout the rest of this section.)

3. Add masses in increments of 50 g, and adjust the frequency so that the same number of nodes is obtained. Take and record measurements for at least six different tensions.

4. The wave speed should be related to the tension F and linear mass density μ by $v = (F/\mu)^{1/2}$. Calculate and record the wave speed in each case using Eq. 14, and plot v^2 as a function of F/μ. (You have calculated v^2 from the frequency and wavelength; these are the y-axis values. You have calculated F/μ from the measured tension and linear mass density; these are the x-axis values. Be sure to convert the tension into units of Newtons.) You now have the experimental points.

5. Now plot the "theoretical" line $v^2 = F/\mu$. This is a straight line at 450 on your graph, if you used the same scale on both axes. Do your experimental and theoretical results agree well? If not, what might be the reasons?

PROCEDURE: PART 3

In this section, we will determine the relationship between frequency and the density of a medium through which a wave propagates.

1. Measure the linear mass densities ($\mu = M/L$) of the two other strings as described above.
2. Keeping the tension and mode number constant at, say, 100 g and $n = 2$, measure and record the frequencies for the three strings.
3. Calculate and record the "experimental" wave speed from the frequency and wavelength for each string density.
4. Calculate and record the "theoretical" wave speed for each string density from $v = (F/\mu)^{1/2}$, and compare these speeds with the experimental values.

DATA

Procedure Part 1:

1. Frequency ($n = 2$ mode) = ______________________________
2. Frequency ($n = 1$ mode) = ______________________________

   Frequency ($n = 2$ mode) = ______________________________

   Frequency ($n = 3$ mode) = ______________________________

   Frequency ($n = 4$ mode) = ______________________________

   Frequency ($n = 5$ mode) = ______________________________

   Frequency ($n = 6$ mode) = ______________________________
3. Wavelength ($n = 1$ mode) = ______________________________

   Wave speed ($n = 1$ mode) = ______________________________

   Wavelength ($n = 2$ mode) = ______________________________

   Wave speed ($n = 2$ mode) = ______________________________

   Wavelength ($n = 3$ mode) = ______________________________

   Wave speed ($n = 3$ mode) = ______________________________

   Wavelength ($n = 4$ mode) = ______________________________

Wave speed ($n = 4$ mode) = ______________________________

Wavelength ($n = 5$ mode) = ______________________________

Wave speed ($n = 5$ mode) = ______________________________

Wavelength ($n = 6$ mode) = ______________________________

Wave speed ($n = 6$ mode) = ______________________________

4. Plot the graph of the frequency as a function of n using one sheet of graph paper at the end of this workbook. Remember to label the axes and title the graph.

Procedure Part 2:

1. Mass of string 1 = ______________________________

   Length of string 1 = ______________________________

2. Frequency (with 50-g mass) = ______________________________

3. Frequency (with 100-g mass) = ______________________________

   Frequency (with 150-g mass) = ______________________________

   Frequency (with 200-g mass) = ______________________________

   Frequency (with 250-g mass) = ______________________________

   Frequency (with 300-g mass) = ______________________________

   Frequency (with 350-g mass) = ______________________________

4. Wave speed (with 50-g mass) = ______________________________

   Wave speed (with 100-g mass) = ______________________________

   Wave speed (with 150-g mass) = ______________________________

   Wave speed (with 200-g mass) = ______________________________

   Wave speed (with 250-g mass) = ______________________________

   Wave speed (with 300-g mass) = ______________________________

   Wave speed (with 350-g mass) = ______________________________

   Plot the experiment graph of v^2 as a function of F/μ using one sheet of graph paper at the end of this workbook. Remember to label the axes and title the graph.

5. Plot the theoretical graph of v^2 as a function of F/μ using the same sheet of graph paper. Remember to label the axes and title the graph.

Procedure Part 3:

1. Mass of string 2 = ______________________________

   Length of string 2 = ______________________________

   Mass of string 3 = ______________________________

   Length of string 3 = ______________________________

2. Frequency (string 1) = ______________________________

   Frequency (string 2) = ______________________________

   Frequency (string 3) = ______________________________

3. Experimental wave speed (string 1) = ______________________________

   Experimental wave speed (string 2) = ______________________________

   Experimental wave speed (string 3) = ______________________________

4. Theoretical wave speed (string 1) = ______________________________

   Theoretical wave speed (string 2) = ______________________________

   Theoretical wave speed (string 3) = ______________________________

   Percentage difference between experimental and theoretical speeds (string 1) = __________

   Percentage difference between experimental and theoretical speeds (string 2) = __________

   Percentage difference between experimental and theoretical speeds (string 3) = __________

ADDITIONAL CREDIT PART 1 (1 mill)

Carefully write out a complete answer to the question posed at the beginning of the experiment: Why does pressing different positions on your guitar string produce different pitches?

____

____

____

ADDITIONAL CREDIT PART 2 (2 mills)

As you tune the frequency, there is a resonance of sorts at each higher mode of vibration. That is, as you tune the frequency, the amplitude of vibration is very large when you are at the correct frequency for the mode, but becomes smaller as you move away from the correct frequency, until you begin to approach the frequency of the next mode. The response might look similar to the graph below.

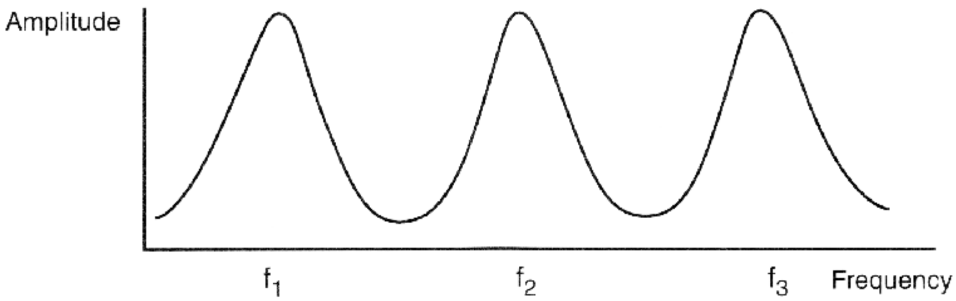


Figure 7

We want to measure the quality factor, Q, of one of these resonances and study how the oscillations decay. Refer to the discussion of Q in Experiment 1.

Choose a mode of vibration where you can get a nice large amplitude (e.g., $n = 2$, with 200 g on the heavy string). Clamp a vertical meter stick near one of the antinodes so you can measure the amplitude of vibration. <u>Measure carefully</u> near the resonance maximum, and record the frequencies on either side of resonance when the amplitude has fallen to $1/\sqrt{2}$ of its maximum value. Using

$$Q = \omega_1/\Delta\omega = f_1/\Delta f, \qquad \text{(Eq. 16 in Experiment 1)}$$

where $f = \omega/2\pi$, determine Q from your measurements.

As discussed in Experiment 1 in connection with its Eq. (16), Q also controls the damping rate of the vibration. Start the wave motion until it builds up to full amplitude. Then switch off the driving vibrator, and observe the wave motion decay. Measure the full amplitude $A_{\max}$ of vibration at resonance by reading off distances from the meter stick while the vibrator is driving the wave, and calculate $A_{\max}/e$ ($e = 2.718\ldots$). <u>Devise a way</u> to note this reduced amplitude on the meter stick.

Start the wave again at full amplitude, switch off the drive, and measure the time required for the amplitude to decay to $A_{\max}/e$ (the so-called "e-folding time"). Compare this time with $2Q/\omega_1 = Q/\pi f_1$, and record the results below.

Amplitude at resonance ($A_{\max}$) = ______________________________

$A_{\max}\sqrt{2}$ = ______________________________

Frequencies at which amplitude is equal to $A_{\max}\sqrt{2}$ = ______________________________

Difference in frequencies (Δf) = ______________________________

Q = ______________________________

$A_{\max}/e$ = ______________________________

Time required for amplitude to decay to $A_{\max}/e$ = ______________________________

$2Q/\omega_1$ = ______________________________

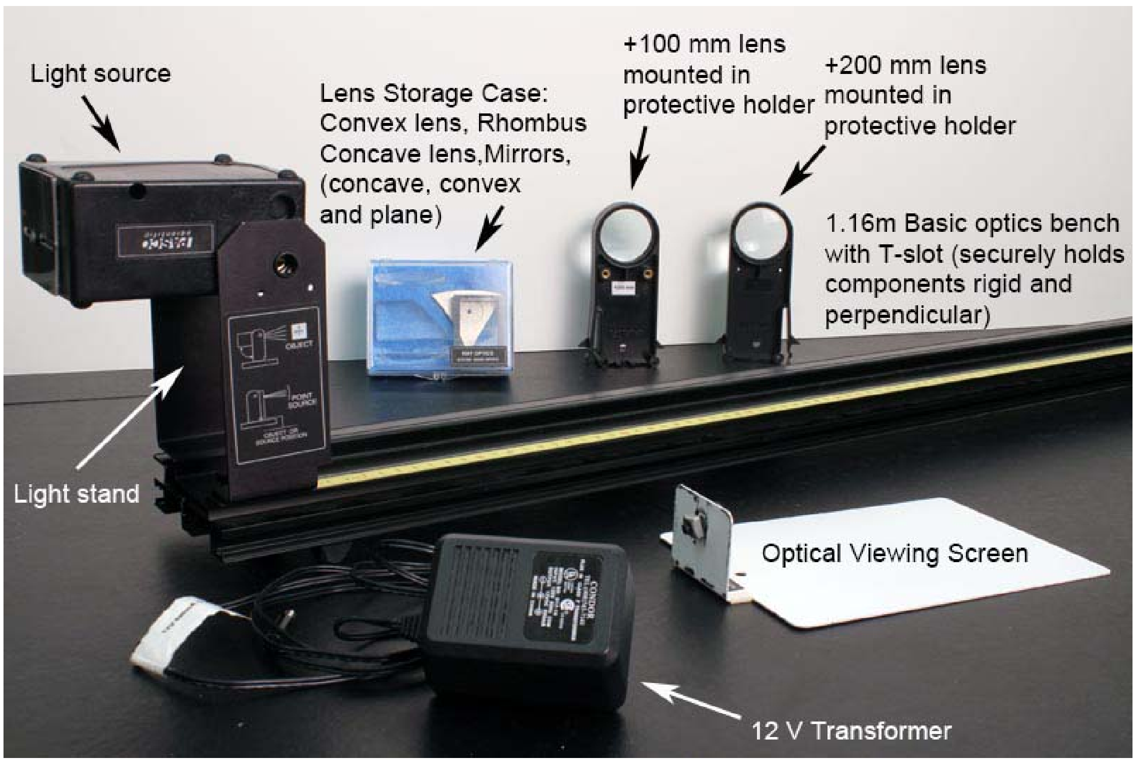
Geometrical Optics

APPARATUS

This lab consists of many short optics experiments. Check over the many pieces of equipment carefully:

Shown in the picture below:

- Optical bench with screen at one end and ray-box bracket at the other end
- Ray box with 12-V transformer
- Lens storage case with four items inside
- Four lens holders with +200 mm, +100 mm, +25 mm, and −25 mm lenses. (Two of these lenses are shown in the diagram below.)



Not shown in the picture above:

- Protractor and ruler
- Diverging lens with unknown focal length
- Card with fine print
- Graph paper
- Tape in room

If anything is missing, notify your TA. At the end of the lab, you must put everything back in order again, and your TA will check for missing pieces.

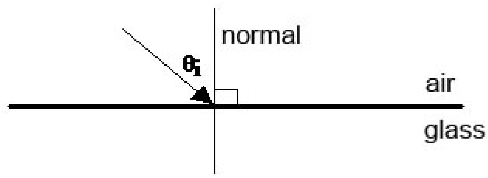
REFLECTION AND REFRACTION

When a beam of light enters a transparent material such as glass or water, its overall speed through the material is slowed from c (3×10^8 m/s) in vacuum by a factor of n (¿ 1):

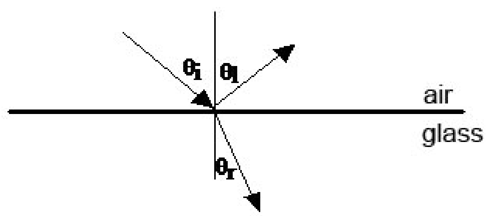
$$\text{(speed in material)} = c/n. \tag{1}$$

The parameter n is called the *index of refraction*, and is generally between 1 and 2 for most transparent materials. Even air has a refractive index slightly greater than 1.

Consider a light beam impinging on the boundary between two transparent materials (e.g., a beam passing from air into glass). By convention, the angle of incidence θ_i is measured with respect to the normal to the boundary.



In general, the beam will be partially reflected from the boundary at an angle θ_l with respect to the normal and partially refracted into the material at an angle θ_r with respect to the normal.



Fermat's Principle, which states that light travels along the path requiring the least time, can be used to derive the laws of reflection and refraction.

<u>Law of Reflection</u>:

$$\theta_\mathrm{l} = \theta_\mathrm{i}. \tag{2}$$

The angle of reflection θ_l is equal to the angle of incidence θ_i.

<u>Law of Refraction</u>:

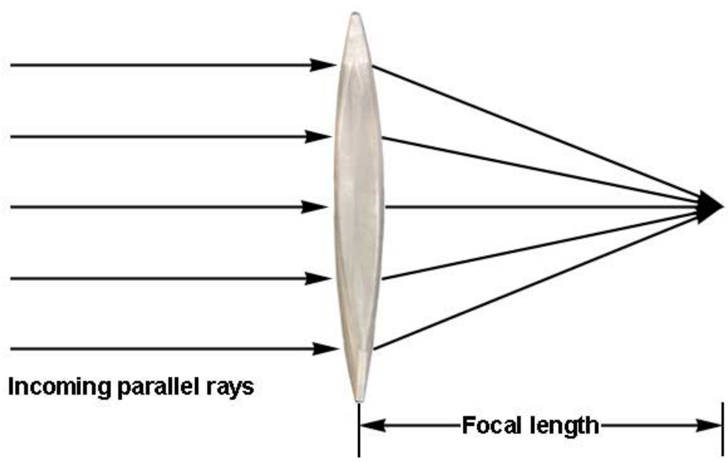
$$n_1 \sin\theta_\mathrm{i} = n_2 \sin\theta_\mathrm{r}. \tag{3}$$

This is also called *Snell's Law*, where n_1 is the refractive index of the material from which light is incident (air in this case), and n_2 is the refractive index of the material to which light is refracted (glass in this case).

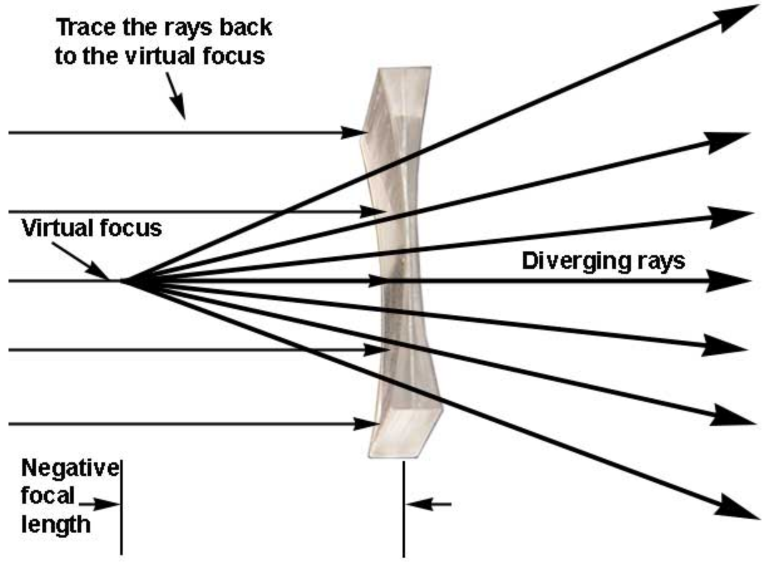
THIN LENSES

A thin lens is one whose thickness is small compared to the other characteristic distances (e.g., its focal length). The surfaces of the lens can be either convex or concave, or one surface could be planar. Because of the refractive properties of its surfaces, the lens will either converge or diverge rays that pass through it. A *converging* lens (such as the first plano-convex lens below) is thicker at its center than at its edges. A *diverging* lens (such as the second concave meniscus lens below) is thinner at its center than at its edges.

If parallel rays (say, from a distant source) pass through the lens, then a converging lens will bring the rays to an approximate focus at some point behind the lens. The distance between the lens and the focus of parallel rays is called the *focal length* of the lens.



If the lens is diverging, then parallel rays passing though the lens will spread out, appearing to come from some point in front of the lens. This point is called the *virtual focus*, and the negative of the distance between the lens and the virtual focus is equal the focal length of the diverging lens.



If an object (say, a lighted upright arrow) is placed near a lens, then the lens will form an image of the arrow at a specific distance from the lens. Let's call the distance between the lens and the object d_{o}, and the distance between the lens and the image d_{i}. Applying the Law of Refraction to the thin lens results in the *thin-lens equation*, which relates these quantities to the focal length f:

$$1/f = 1/d_{\text{o}} + 1/d_{\text{i}}. \tag{4}$$

Recall that f can be positive or negative, depending on whether the lens is converging or diverging, respectively. Once the object distance d_{o} is chosen, the image distance d_{i} may turn out to be positive or negative. If d_{i} is positive, then a *real* image is formed. A real image focuses on a screen located a distance d_{i} behind the lens. If d_{i} is negative, then a *virtual* image is formed. A virtual image does not focus anywhere, but light emerges from the lens as though it came from an image located a distance $\mid d_{\text{i}} \mid$ in front of the lens. You can see the virtual image by looking back through the lens toward the object. Such an image can be observed when you are looking through a diverging lens. These virtual images looks smaller and more distant.

CURVED MIRRORS

A curved mirror can also converge or diverge light rays that impinge on it. A converging mirror is concave, while a diverging mirror is convex. The mirror equation is identical to the thin-lens equation:

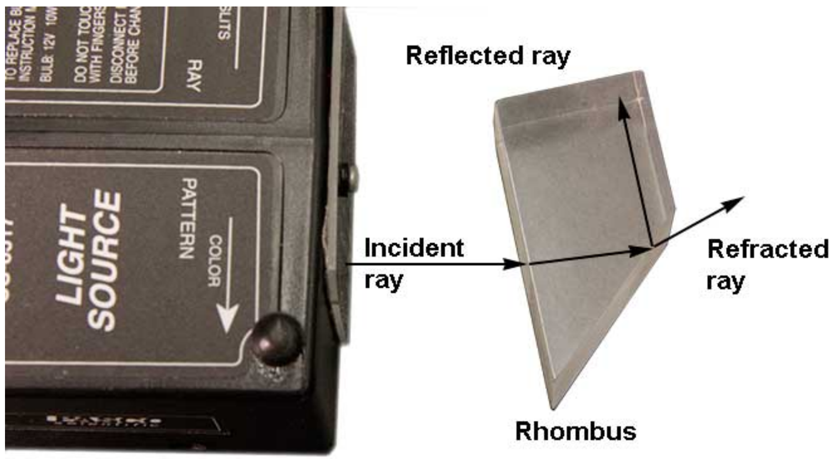
$$1/f = 1/d_{\text{o}} + 1/d_{\text{i}}. \tag{5}$$

We just need to remember that a real image with a positive image distance d_{i} will be formed on the same side of the mirror as the incident rays from the object, while a virtual image with a negative image distance d_{i} will be formed behind the mirror.

PROCEDURE PART 1: REFRACTION AND TOTAL INTERNAL REFLECTION

1. Place the ray box, label side up, on a white sheet of paper on the table. Plug in its transformer. Adjust the box so that one white ray is showing.

2. Position the rhombus as shown in the figure. The triangular end of the rhombus is used as a prism in this experiment. Keep the ray near the point of the rhombus for the maximum transmission of light. Notice that a refracted ray emerges from the second surface, and a reflected ray continues in the acrylic of the rhombus.



3. The incident ray is bent once as it enters the acrylic of the rhombus, and again as it exits the rhombus. Vary the angle of incidence. Does the exiting ray bend toward or away from the normal? (Physicists and opticians measure the angles of the rays with respect to the *normal*, a line perpendicular to the surface.)

   Does the exiting ray bend toward or away from the normal? ____________________________

4. Pick an angle of incidence for which the exiting ray is well bent, and trace neatly the internal and exiting rays on the top half of the paper underneath. Also trace the rhombus-air interfaces, clearly marking the side corresponding to the rhombus and that corresponding to air. You can simply mark the ends of the rays and use a ruler to extend the rays. Use the protractor to construct the normal to the interface and measure the angles of the two rays with respect to the normal. With these angles, use Snell's Law to find the refractive index of the acrylic of the rhombus. (Use $n = 1$ for air.)

   Angle of ray in acrylic = ______________________________

   Angle of ray in air = ______________________________

   Refractive index (n) of acrylic = ______________________________

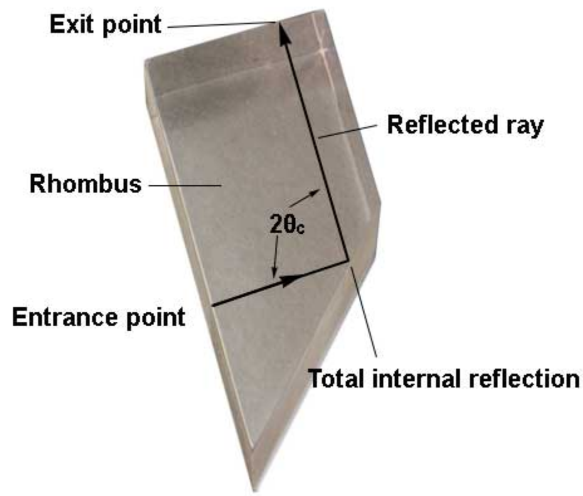
   Show your calculation of n below:

   ____

   ____

____

5. Total internal reflection: Rotate the rhombus until the exiting ray travels parallel to the surface (separating into colors), and then rotate the rhombus slightly farther. Now there is no refracted ray; the light is totally internally reflected from the inner surface. *Total internal reflection* occurs only beyond a certain "critical angle" θ_c, the angle at which the exiting refracted ray travels parallel to the surface. Rotate the rhombus again, and notice how the reflected ray becomes brighter as you approach and reach the critical angle. When there is both a refracted ray and a reflected ray, the incident light energy is divided between these rays. However, when there is no refracted ray, all of the incident energy goes into the reflected ray (minus any absorption losses in the acrylic).



Adjust the rhombus exactly to the critical angle, and trace neatly the ray in the acrylic and the refracting surface on the bottom half of the paper. Construct the normal to the surface, and measure the critical angle of the ray. (Again, all angles are measured with respect to the normal.) According to the textbook, the sine of the critical angle is

$$\sin\theta_c = 1/n. \tag{6}$$

Calculate n from this relation, and compare it to the n determined in step 4.

Measured critical angle θ_c = ______________________________

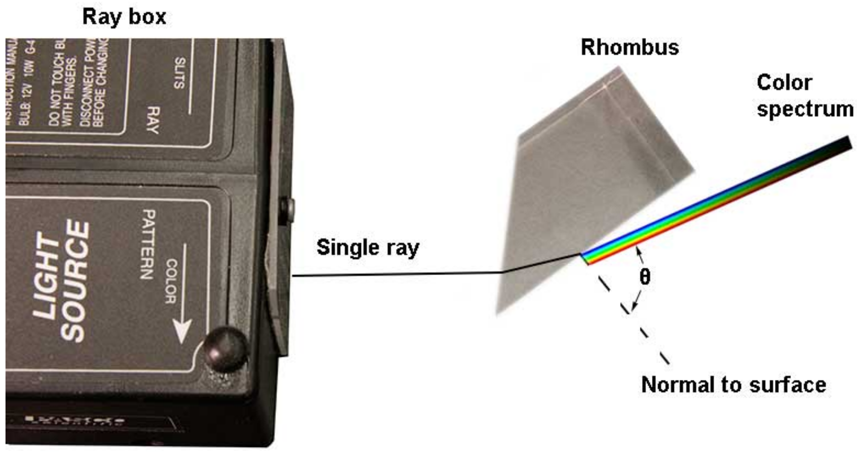
Refractive index (n) determined from critical angle = ______________________________

Refractive index (n) determined from Snell's Law (copy from step 4) = ____________________

6. Adjust the rhombus until the angle of the exiting ray is as large as possible (but less than the critical angle) and still clearly visible, and the exiting ray separates into colors. This phenomenon is called *dispersion* and illustrates the refraction of different colors at various angles. Which color is refracted at the largest angle, and which color is refracted at the smallest angle?

Color refracted at largest angle = ______________________________

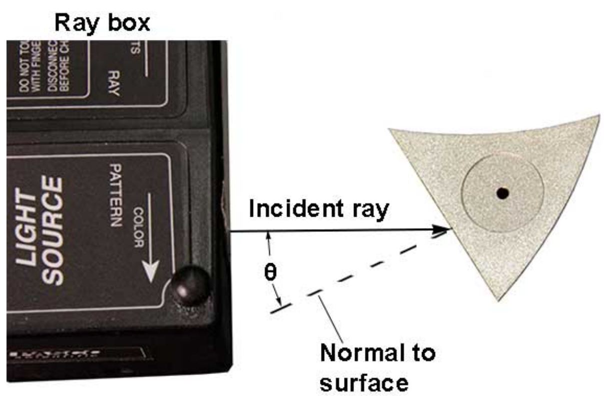
Color refracted at smallest angle = ______________________________



PROCEDURE PART 2: REFLECTION

1. As in the preceding section, the ray box should be on a white sheet of paper, label side up, with one white ray showing.

2. Place the triangular-shaped mirror piece on the paper, and position the plane surface so that both the incident and reflected rays are clearly seen.



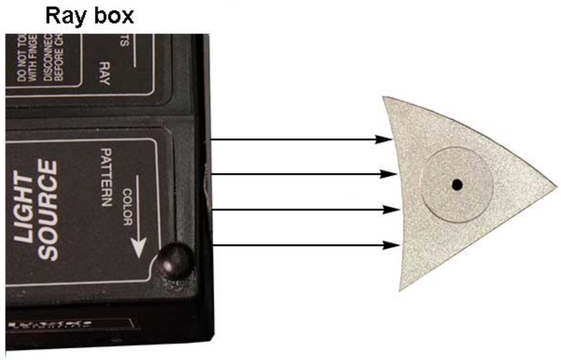
3. By turning the mirror piece, vary the angle of incidence while observing how the angle of reflection changes. What is the relation between the angle of incidence and the angle of reflection?

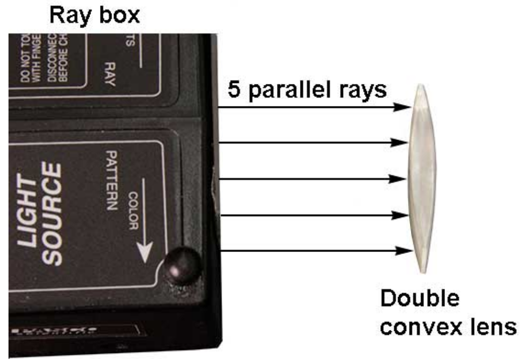
   Relation:

   ____

PROCEDURE PART 3: CONVERGENCE AND DIVERGENCE OF RAYS

1. Either a mirror or a lens can converge or diverge parallel rays. The triangular mirror piece has a concave and a convex side, and there is a section of a double convex lens and a double concave lens. Adjust the ray box so that it makes five parallel white rays.

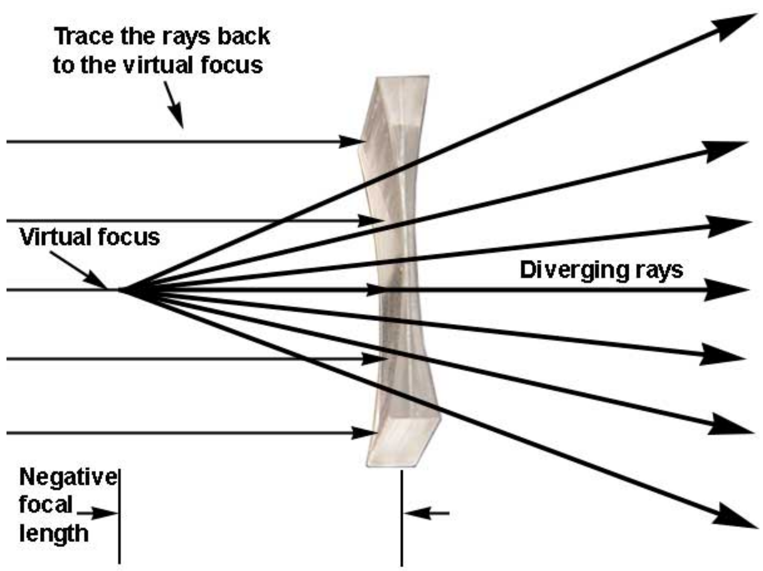




2. The concave mirror and the double convex lens (shown above) both converge the parallel rays to an approximate *focal point.* The distance between the lens or mirror surface and the focal point of parallel rays is the *focal length* of the mirror or lens. Measure the focal lengths of the concave mirror and convex lens in centimeters, and enter them in the table below.

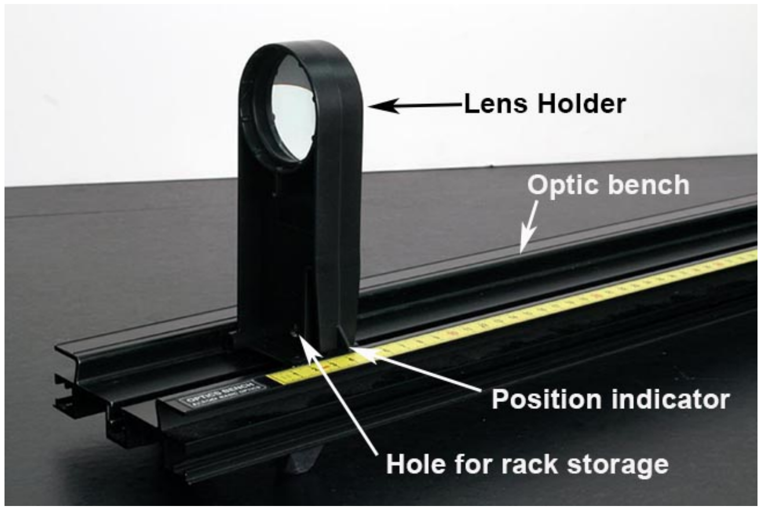
	A	B
1	Optical element	Focal Length
2	Concave mirror	
3	Double convex lens	
4	Convex mirror	
5	Double concave lens	

3. The convex mirror and the double concave lens both diverge the parallel rays. They have negative focal lengths, and the magnitude of the focal length is equal to the distance between the optical element and the point from which the rays appear to diverge. Using the convex mirror and the double concave lens (one at a time), sketch the mirror or lens surface in position, and trace the diverging rays on the white paper. Remove the mirror or lens, and continue tracing the rays back to the *virtual focus* using a ruler. Enter the (negative) focal lengths (in centimeters) of these optical elements in the table above.



PROCEDURE PART 4: IMAGE FORMATION AND FOCAL LENGTH OF A LENS

1. Place the 200-mm lens and the screen on the optical-bench track. Do not put the light source on yet.



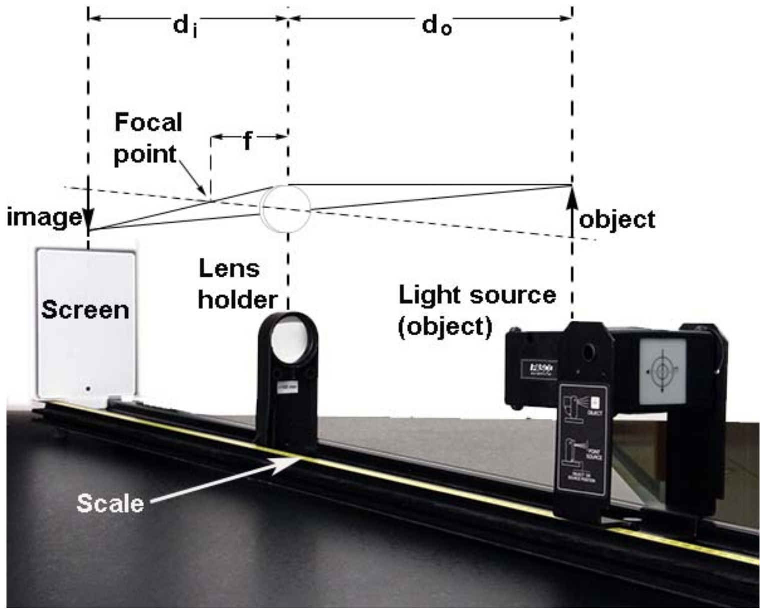
2. Focus a distant light source (such as a window, the trees outside the window, or a light at the other end of the room) on the screen. A distant source is effectively at infinity, the rays from the source are parallel, and the lens converges the rays to an image at the focal point. Measure the distance between the lens and the screen, and compare this distance to the stated focal length. (You may read positions off the optical bench scale and subtract them to find distances.)

   Notice that the image is inverted. This is similar to how your eye lens forms an inverted image of the outside world on your retina.

3. Now mount the light source with the circles and arrows side facing the lens and screen. You need to unplug the power cord of the light box, and then replug it when the box is mounted. (There are two ways to mount the light source in the bracket. Notice the two holes in the bracket for the detent buttons on either side. For one way, the offset of the bracket arm below permits reading the position of the light source directly from the scale; for the other way, you would need to correct for the setback of the light source.)

4. Adjust the position of the lens until the image of the light source is focused sharply on the screen. Read the distances d_o and d_i in the figure off the scale, and calculate the focal length of the lens from the thin-lens equation:

$$1/f = 1/d_\mathrm{o} + 1/d_\mathrm{i}. \tag{7}$$



Enter the data below:

d_o = ______________________________

d_i = ______________________________

$1/f$ = ______________________________

f (calculated) = ____________________

f (theoretical) = ____________________

Here f (calculated) is the value obtained from the thin-lens equation, and f (theoretical) is the value read off the lens. These two focal lengths should, of course, agree approximately.

PROCEDURE PART 5: TWO-LENS SYSTEMS

1. Place the light source at 110 cm and the screen at 60 cm on the optical-bench scale. Place the +100 mm lens between the light source and the screen. You will find it possible to obtain a sharp image on the screen with the lens at about 72 cm. Is the image upright or inverted? Adjust for the exact focus, measure the object and image distances (read positions off the scale and subtract them), and compare the theoretical focal length with that obtained from the thin-lens equation.

   d_o = ______________________________

   d_i = ______________________________

   $1/f$ = ______________________________

   f (calculated) = ____________________

f (theoretical) = ______________________________

Is the image upright or inverted? ______________________________

2. The image formed at 60 cm can serve as the object for a second lens. Move the screen back, and place the +25 mm lens at 55 cm on the scale. Where does the image focus? Is it upright or inverted? Measure the object and image distances for the +25 mm lens, and compare the theoretical focal length with that obtained from the thin-lens equation.

   d_o = ____

   d_i = ____

   $1/f$ = ____

   f (calculated) = ______________________________

   f (theoretical) = ______________________________

   Is the image upright or inverted? ______________________________

3. If a second lens is placed <u>inside</u> the focal point of the first lens, the image of the first lens serves as a *virtual object* for the second lens. Place the +200 mm lens at 68 cm on the scale. The object distance is the negative of the distance between the lens and the point at which the image would have formed: namely, −8.0 cm if the first image were at 60 cm and the second lens were at 68 cm. Where does the image focus now? Is it upright or inverted? Compare the theoretical focal length with that obtained from the thin-lens equation.

   d_o = ____

   d_i = ____

   $1/f$ = ____

   f (calculated) = ______________________________

   f (theoretical) = ______________________________

   Is the image upright or inverted? ______________________________

4. Repeat step 3, placing the diverging −25 mm lens at 68 cm on the scale. This strongly diverging lens bends the rays from the +100 mm lens outward so that they diverge and never come to a focus beyond the lens. Instead, look through the two lenses back to the source. You will see the virtual image at a distance. Is it upright or inverted? Calculate the image distance of the −25 mm lens with its virtual object. The result comes out negative. Look through the lenses again. Does the image distance seem reasonable?

   d_o = ____

   d_i = ____

   Does image distance seem reasonable? ______________________________

Is the image upright or inverted? ______________________________

PROCEDURE PART 6: SIMPLE TELESCOPES

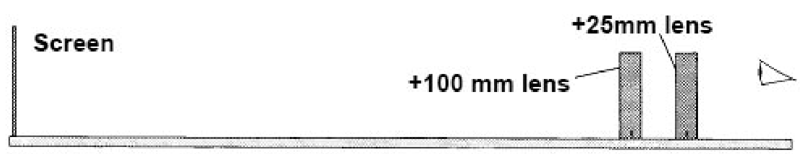
1. You have four lenses in holders; the lenses have focal lengths of +200 mm, +100 mm, +25 mm, and −25 mm. Carry these lenses over to a window so that you can look out at distant objects (such as a building across the quadrangle). DO NOT LOOK AT THE SUN WITH YOUR TELESCOPE ARRANGEMENTS. PERMANENT EYE DAMAGE MAY RESULT. To make a telescope, hold one of the short focal-length lenses near your eye and one of the longer focal-length lenses out with your arm, so that you look through both lenses in series. Adjust the position of the second lens (the objective) until a distant object is focused. The lens nearest your eye is called the *eye lens*, and the one farther out is called the *objective*.

2. Galilean telescope: Use the negative focal-length lens ($f = -25$ mm) as the eye lens and the +100 mm or +200 mm lens as the objective, and focus a distant object. Notice that the field of view is small and the image is distorted. Nevertheless, Galilei used an optical arrangement similar to this to discover the moons of Jupiter, the phases of Venus, sunspots, and many other heavenly wonders.

3. Astronomical telescope: Use the +25 mm lens as the eye lens and the +100 mm or +200 mm lens as the objective, and focus a distant object. Notice that the field of view is now larger and the image is sharper, although the image is inverted. You can also try the +100 mm lens as the eye lens and the +200 mm lens as the objective.

PROCEDURE PART 7: MEASURING THE POWER OF AN ASTRONOMICAL TELESCOPE

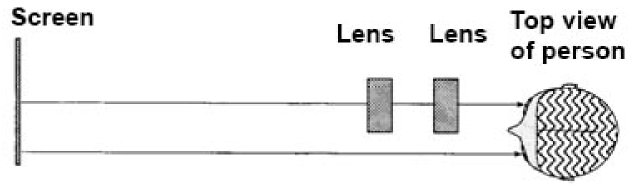
1. Use the +25 mm lens as the eye lens and the +100 mm lens as the objective. Place the lenses near one end of the optical bench and the screen at the other end, as shown below. Tape a piece of graph paper to the screen. (Graph paper and tape are in the lab room.)



2. Look through the eye lens, and focus the image of the graph paper by moving the objective.

3. (This procedure is a bit complex. Try your best and do not waste a lot of time on it.) Eliminate parallax by moving the eye lens until the image is in the same plane as the object (the screen). To observe the parallax, open both eyes and look through the lens at the image with one eye, while looking around the edge of the lens directly at the object with the other eye. Refer to the figures below. The lines of the image (solid lines in the figure below) will be superimposed on the lines of the object (dotted lines in the figure below). Move your head back and forth, and up and down. As you move your head, the lines of the image will move relative to the lines of the object due to parallax. To eliminate parallax, adjust the eye lens until the image lines do not move relative to the object lines when you move your head. When there is no

parallax, the lines in the center of the lens appear to be stuck to the object lines. (Even when there is no parallax, the lines may appear to move near the edge of the lens because of lens aberrations.)



4. Measure the magnification of this telescope by counting the number of squares in the object that lie along a side of one square of the image. To do this, you must view the image through the telescope with one eye, while looking directly at the object with the other eye. Record the observed magnification in step 5.

5. The theoretical magnification for objects at infinity is equal to the ratio of the focal lengths. Record and compare the theoretical and observed magnifications below.

   Observed magnification = ______________________________

   Theoretical magnification = ______________________________

ADDITIONAL CREDIT PART 1: MEASURING A GLASSES PRESCRIPTION (3 mills)

The inverse of the focal length of a lens, $P = 1/f$, is called the *power* of the lens. The units of power are inverse meters which are renamed *diopters*, a unit commonly used by optometrists and opticians. The larger the power, the more strongly the lens converges rays. You can show that when two thin lenses are placed close together (so that the distance between them is much less than the focal lengths), the power P_{T} of the combined lenses is the sum of the powers P_1 and P_2 of the individual lenses:

$$P_{\mathrm{T}} = P_1 + P_2 \tag{8}$$

or

$$1/f_{\mathrm{T}} = 1/f_1 + 1/f_2. \tag{9}$$

The closest distance at which you can focus your eyes clearly (when you are exerting maximum muscle tension on your eye lens) is called your *near point.* The farthest distance at which you can focus your eyes clearly (when your focusing muscles are relaxed) is called your *far point.* Ideally, your far point is at infinity, and your near point is at least as small as 25 cm so you can read easily. If you are nearsighted, then your far point is at some finite distance; you cannot focus distant objects clearly. If you are farsighted, then your far point is “beyond infinity”, so to speak, so that you need to exert eye-lens muscle tension even to focus distant objects. As you grow older, your *power of accommodation* (i.e., your ability to change the focal length of your eye lens) weakens and your near point moves out, so that you must have corrective lenses to focus on close objects, such as for reading. Thus, you notice older persons wearing reading glasses.

You may be wearing glasses, contact lenses, or have had laser eye surgery to correct your vision — or you may be lucky and have "perfect" vision without correction. In any case, use a meter stick (or other ruler) and the card with fine print to measure your near point (with correction, if any) as in the illustration below. The purpose of laying the meter stick on the table is to avoid poking it toward your eye.

Move the card in to the closest distance that you can focus clearly.

Near-point distance (corrected) = ______________________________

If you are nearsighted and wearing glasses, take off your glasses and measure your far point. (If you are wearing contacts, you may remove a contact and try this, but the step is optional.)

Far-point distance (uncorrected) = ______________________________

If you or your lab partner are nearsighted and wearing glasses, determine your glasses prescription as instructed below. If neither you nor your partner is nearsighted and wearing glasses, use the (uncorrected) data for Dr. Art Huffman: far point = 20 cm, near point = 18 cm. (Yes, his vision is that bad!)

If the eye is nearsighted, we want to put a diverging lens in front of it, which will shift the uncorrected far point to infinity. We can use the formula above to find the focal length of the glasses-eye combination. Let the (uncorrected) far-point distance be d, the eye-to-retina distance be i, the focal length of the eye lens while relaxed be f_{f}, and the focal length of the glasses be f_{g}:

$$\begin{aligned} \text{Without glasses:} \quad & 1/d + 1/i = 1/f_{\mathrm{f}} & (10) \\ \text{With glasses:} \quad & 1/\infty + 1/i = 1/f_{\mathrm{f}} + 1/f_{\mathrm{g}}. & (11) \end{aligned}$$

Subtracting the first equation from the second gives the power P_{g} of the glasses:

$$P_{\mathrm{g}} = 1/f_{\mathrm{g}} = -1/d. \tag{12}$$

Compute your glasses prescription (or Art's) in diopters:

____

ADDITIONAL CREDIT PART 2: MEASURING THE FOCAL LENGTH OF A DIVERGING LENS (2 mills)

Devise a way, using your optical bench, to measure the focal length of a diverging lens. Then measure the focal length of your glasses as in Additional Credit Part 1, or measure the focal length of one of the unknown lenses supplied in the lab.

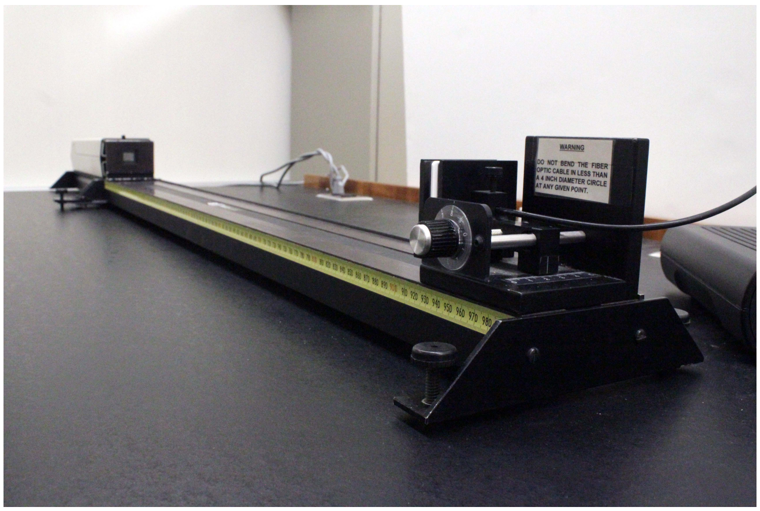
Sketch your plan for measuring the focal length of a diverging lens below, and report the measured power of the glasses or of the unknown lens.

Physical Optics

APPARATUS

Shown in the picture below:

- Optics bench with laser alignment bench and component carriers
- Laser
- Linear translator with photometer apertures slide and fiber optic cable



Not shown in the picture above:

- Computer with ScienceWorkshop interface
- High sensitivity light sensor with extension cable
- Slit slides and polarizers
- Incandescent light source
- Tensor Lamp

INTRODUCTION

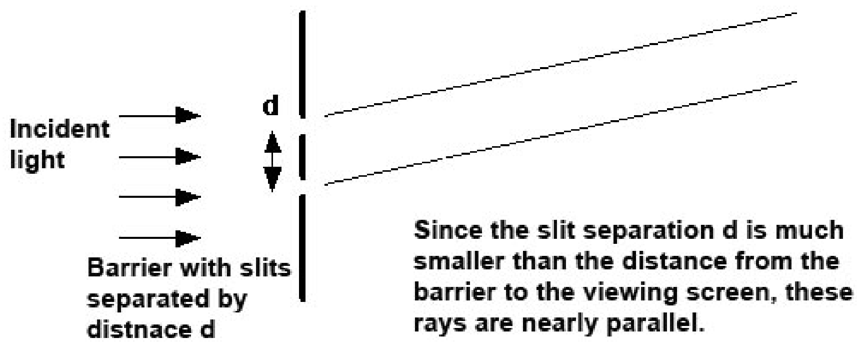
The objective of this experiment is to familiarize the student with some of the amazing characteristics of a laser, such as its coherence and small beam divergence. The laser will be used to investigate single- and double-slit diffraction and interference, as well as polarization. Furthermore, several interesting diffraction phenomena that are hard to see with standard light sources can be observed easily with the laser.

WARNING: Do not look directly into the laser beam! Permanent eye damage (a burned spot on the retina) may occur from exposure to the direct or reflected beam. The beam can be viewed without any concern when it is scattered from a diffuse surface such as a piece of paper. The laser beam is completely harmless to any piece of clothing or to any part of the body except the eye.

It is a wise precaution to keep your head well above the laser-beam height at all times to avoid accidental exposure to your own or your fellow students' beams. Do not insert any reflective surface into the beam except as directed in the instructions or as authorized by your TA. The laser contains a high-voltage power supply. Caution must be used if an opening is found in the case to avoid contacting the high voltage. Report any problems to your TA.

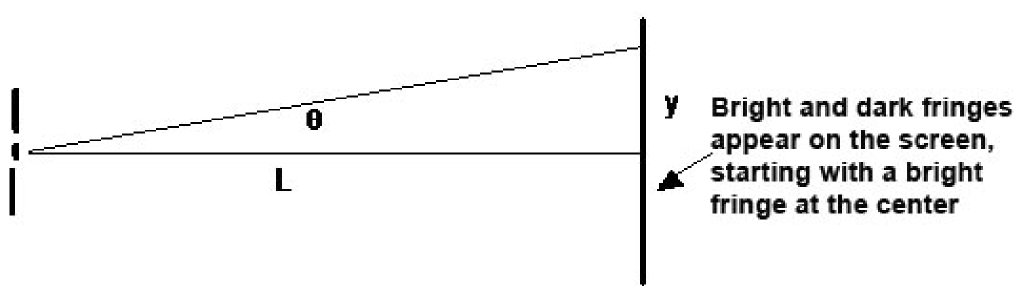
DOUBLE-SLIT INTERFERENCE

In the first part of the experiment, we will measure the positions of the double-slit interference minima. Schematically, a double-slit setup looks as follows:

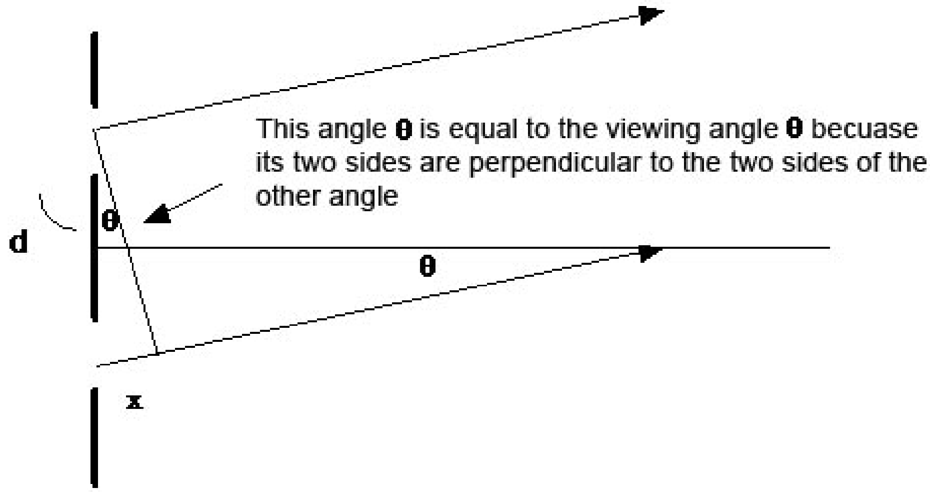


The incident laser light shining on the slits is *coherent*: at each slit, the light ray starts with the same phase. To reach the same point on the viewing screen, one ray needs to travel slightly farther than the other ray, and therefore becomes out of phase with the other ray. If one ray travels a distance equal to one-half wavelength farther than the other way, then the two rays will be 180° out of phase and cancel each other, resulting in destructive interference. No light reaches this point on the screen. At the center of the screen, the two rays travel exactly the same distance and therefore interfere constructively, producing a bright fringe. At a certain distance from the center of the screen, the rays will be one-half wavelength (or 180°) out of phase with each other and interfere destructively, producing a dark fringe. A bit farther along the screen, the rays will be one whole wavelength (or 360°) out of phase with each other and interfere constructively again. Farther still, the ways will be one and one-half wavelengths (or 360° + 180° = 540°) out of phase with each other and interfere destructively. Thus, the interference pattern contains a series of bright and dark fringes on the screen.

Let θ be the viewing angle from the perpendicular, as shown in the figure below:



Study the construction in Figure 3.



The small extra distance x that the lower ray needs to travel is $d \sin\theta$. If this distance is equal to an odd multiple of one-half wavelength, then the two rays will interfere destructively, and no light will reach this point on the screen:

$$\begin{aligned} &\text{interference minima at} \\ d\sin\theta &= N(\lambda/2) \\ &\text{for } N = 1, 3, 5, \ldots \end{aligned} \tag{1}$$

(An even value of N would separate the two rays by a whole number of wavelengths, causing them to interfere constructively.) Now, note that the expression $N = 2(n + 1/2)$ reproduces the odd numbers $N = 1, 3, 5, \ldots$ for $n = 0, 1, 2, \ldots$, so we can rewrite Eq. 1 as:

$$\begin{aligned} &\text{interference minima at} \\ d\sin\theta &= (n + 1/2)\lambda \\ &\text{for } n = 0, 1, 2, \ldots. \end{aligned} \tag{2}$$

Look at Figure 2 again. If y is the linear distance from the center of the pattern on the screen to the point of interference, and if the angle θ is small, then $\sin\theta \approx \tan\theta = y/L$. Thus, the positions of the minima are given by

$$y_n = (n + 1/2)\lambda L/d \quad \text{for } n = 0, 1, 2, \ldots, \tag{3}$$

and the distance between successive minima is

$$\Delta y = (y_{n+1} - y_n) = \lambda L/d. \tag{4}$$

The first part of this experiment involves measuring the positions of the interference minima and determining the wavelength of the laser light.

SINGLE-SLIT DIFFRACTION

When light passes through a single slit of non-zero width, rays from the different parts of the slit interfere with one another and produce another type of interference pattern. This type of interference — in which rays from many infinitesimally close points combine with one another — is called *diffraction*. We will measure the actual intensity curve of a diffraction pattern.

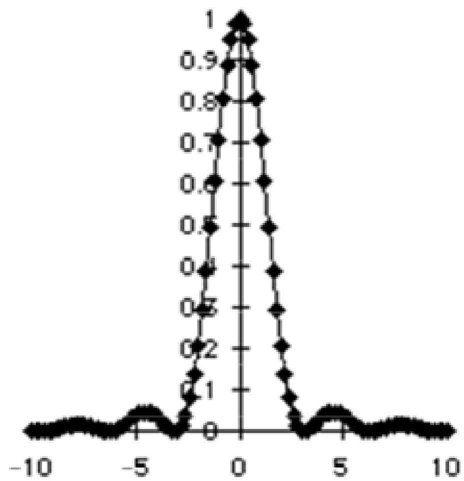
The textbook or the appendix to this experiment gives the derivation of the intensity curve of the diffraction pattern for a single slit:

$$I = I_0[(\sin\alpha)/\alpha]^2, \tag{5}$$

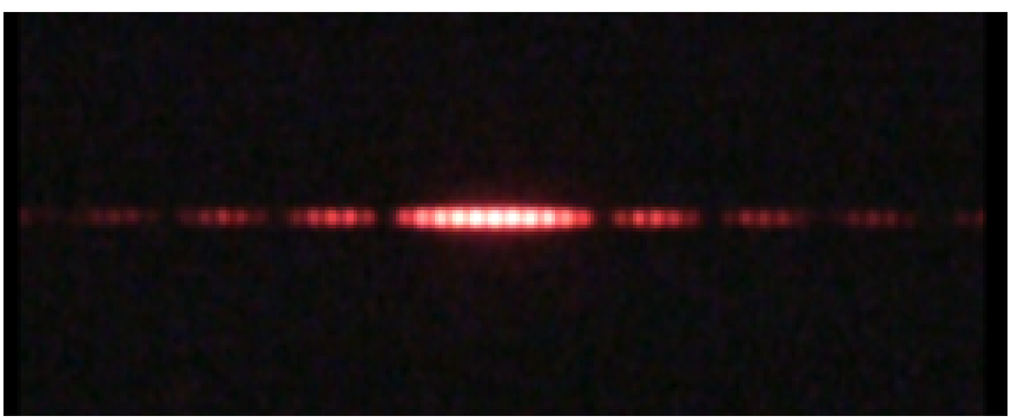
where

$$\alpha = \pi a \sin\theta/\lambda, \tag{6}$$

a is the slit width, and θ is the viewing angle. Here is a plot of the intensity I from Excel:



The image below demonstrates the intensity pattern; it shows the broad central maximum and much dimmer side fringes.



Let us locate the minima of the single-slit diffraction pattern. From Eq. 5, the minima occur where $\sin\alpha = 0$, except where α itself is zero. When α is zero, $\sin\alpha = 0$, and the expression $0/0$ is indeterminate. L'Hopital's rule resolves this ambiguity to show that $\sin\alpha/\alpha \to 1$ as $\alpha \to 0$. Thus, $\alpha = 0$ corresponds to the center of the pattern and is called the central maximum. Elsewhere, the denominator is never zero, and the minima are located at the positions $\sin\alpha = 0$ or $\alpha = n\pi$, with $n =$ any integer <u>except</u> 0. From Eq. 6, we find that:

$$\begin{gathered}\text{diffraction minima at}\\ a\sin\theta = n\lambda\\ \text{for } n = \text{any integer } \underline{\text{except}}\ 0.\end{gathered} \tag{7}$$

Note that the central maximum is twice as wide as the side fringes. The centers of the side fringes are located approximately (but not exactly) halfway between the minima where $\sin\alpha$ is either $+1$ or -1, or $\alpha = (n + 1/2)\pi$, with $n =$ any integer <u>except</u> 0:

$$\begin{gathered}\text{diffraction maxima approx. at}\\ a\sin\theta = (n + 1/2)\lambda\\ \text{for } n = \text{any integer } \underline{\text{except}}\ 0.\end{gathered} \tag{8}$$

(The maxima are only approximately at these positions because the denominator of Eq. 5 depends on α. To find the exact positions of the maxima, we need to take the derivative of I with respect to α and set it equal to zero, then solve for α.)

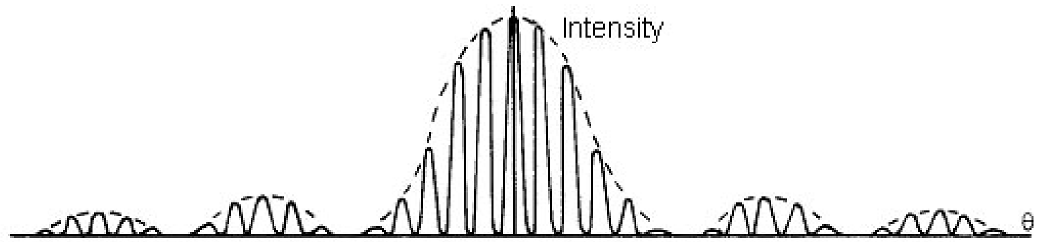
As mentioned above, the side fringes are much dimmer than the central maximum. We can estimate the brightness of the first side fringe by substituting its approximate position $\alpha = 3\pi/2$ into Eq. 5:

$$I(\text{first side fringe})/I_0 = 1/(3\pi/2)^2 = 0.045. \tag{9}$$

The first side fringe is only 4.5% as bright as the central maximum.

Note an important point about diffraction: As the single slit is made more narrow, the central maximum (and indeed the entire pattern) spreads out. We can see this most directly from the position of the first minimum in Eq. 7: $\sin\theta_1 = \lambda/a$. As we try to "squeeze down" the light, it spreads out instead.

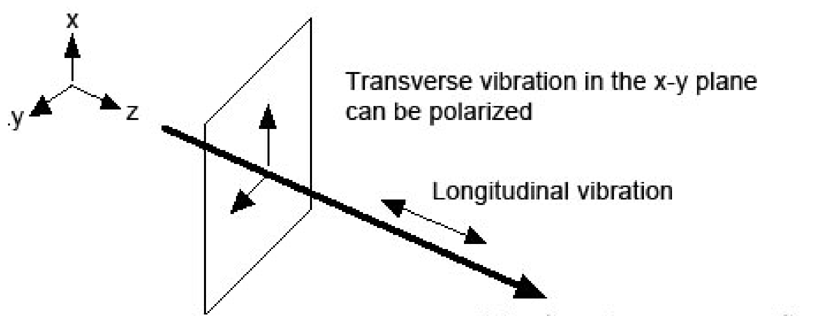
Consider the double-slit interference setup again. Eq. 3 shows that the fringes are equally spaced for small viewing angles, but we now wish to determine the brightness of the fringes. If the two slits were very narrow — say, much less than a wavelength of light ($a \ll \lambda$) — then the central maxima of their diffraction patterns would spread out in the entire forward direction. The interference fringes would be illuminated equally. But we cannot make the slits too narrow, as insufficient light would pass through them for us to see the fringes clearly. The slits must be of non-zero width. Their central diffraction maxima will nearly overlap and illuminate the central area of the interference fringes prominently, while the side fringes of the diffraction pattern will illuminate the interference fringes farther from the center. A typical example is shown below.



The single slit diffraction pattern modulates the double slit interference pattern

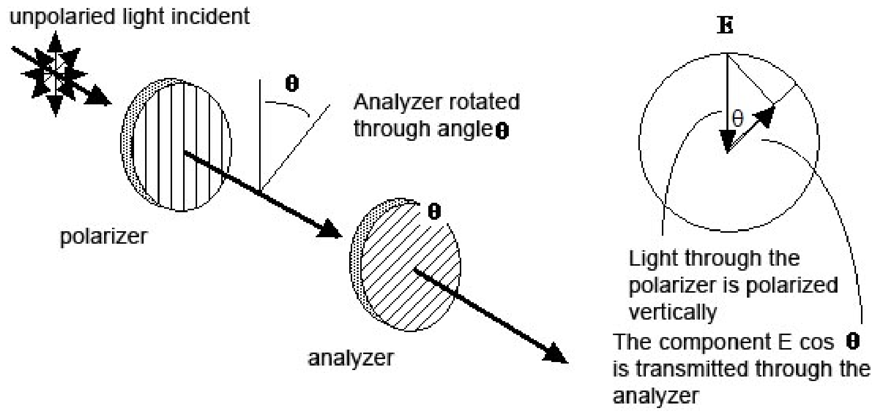
POLARIZATION

Consider a general wave moving in the z direction. Whatever is vibrating could be oscillating in the x, y, or z directions, or in some combination of the three directions. If the vibration is along the direction of wave motion (i.e., in the z direction), then the wave is said to be longitudinal. Sound is a *longitudinal* wave of alternate compressions and rarefactions of air. If the vibration is perpendicular to the direction of wave motion (i.e., in the xy plane), then the wave is said to be transverse. (Certain kinds of waves are neither purely longitudinal nor transverse.) Since one particular direction within the xy plane can be selected, a transverse wave can be *polarized*.



The simple fact that light can be polarized tells us that light is a transverse wave. According to Maxwell's equations, light is electromagnetic radiation. The electric and magnetic field vectors oscillate at right angles to each other and to the direction of wave propagation. We assign the direction in which the electric field oscillates as the polarization direction of light. The light from typical sources such as the Sun and light bulbs is *unpolarized*; it is emitted from many different atoms vibrating in random directions. A simple way to obtain polarized light is to filter unpolarized light through a sheet of Polaroid. Such a sheet contains long, asymmetrical molecules which have been cleverly arranged so that the axes of all molecules are parallel and lie in the plane of the sheet. The long Polaroid molecules in the sheet are all oriented in the same direction. Only the component of the incident electric field perpendicular to the axes of the molecules is transmitted; the component of the incident electric field parallel to the axes of the molecules is absorbed.

Consider an arrangement of two consecutive Polaroid sheets:



The first sheet is called the *polarizer*, and the second one is called the *analyzer*. If the axes of the polarizer and analyzer are crossed (i.e., at right angles to each other), then no light passes through the sheets. (Real polarizers are not 100% efficient, so we might not see exactly zero light.) If the axis of the analyzer were aligned parallel to that of the polarizer, then 100% of the light passing the polarizer would be transmitted through the analyzer. The diagram above shows that if the analyzer is oriented at an angle θ with respect to the polarizer, then a component of the incident electric field $E\cos\theta$ will be transmitted. Since the intensity of a wave is proportional to the square of its amplitude, the intensity of light transmitted through two polarizers at an angle θ with respect to each other is proportional to $\cos^2\theta$. This result is called *Malus' Law*, which we will test in this experiment.

An interesting situation arises if a third polarizer is inserted between two crossed polarizers. No light passes through the crossed polarizers initially, but when the third polarizer is added, light is able to pass through when the third polarizer has certain orientations. How can the third polarizer, which can only absorb light, cause some light to pass through the crossed sheets?

EQUIPMENT

At your lab station is an optics bench. A laser is located at one end of the bench, on a laser alignment bench, while a linear translator with a dial knob that moves the carriage crossways on the bench can be found at the other end. Between the laser and the linear translator are one or more movable component carriers. Fitted into a small hole in the linear translator is a fiber-optic probe connected to a high-sensitivity light sensor which, in turn, is connected by an extension cable to the ScienceWorkshop interface. Be careful with the probe. Do not bend the probe in a circle of less than 10-cm diameter at any given point. Also, do not bend the probe within 8 cm of either end. A slit of width 0.2 mm has been placed just in front of the probe to provide 0.2-mm resolution. (Note: Do not remove the Photometer Apertures slide from the translator and use it as a single slit.)

Light from the laser is transmitted through the probe to the high sensitivity light sensor, which provides an intensity reading. The linear translator (which is basically a carriage mounted on a threaded rod) moves the probe along the axis of the rod. An intensity plot of the pattern produced by a slit placed between the light source and the probe can be made by scanning the probe along the axis of the rod and taking readings from the high sensitivity light sensor.

The probe can be attached to the high sensitivity light sensor by slipping the optic output connector (BNC plug) of the probe over the input jack on the high sensitivity light sensor. A quarter-twist clockwise locks the probe to the high sensitivity light sensor; push the connector towards the sensor box and a quarter-twist counterclockwise disengages it.

The probe attenuates the light intensity reaching the selenium cell to approximately 6.5% of its value when the probe is not used. This makes measurements of absolute intensity impossible. However, for these experiments, only the relative intensities are needed.

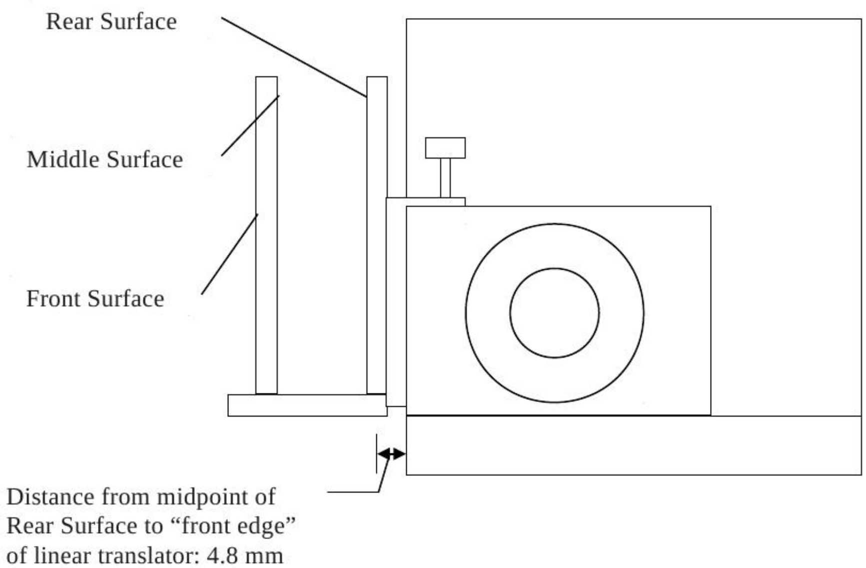
PROCEDURE PART 1: DOUBLE-SLIT INTERFERENCE

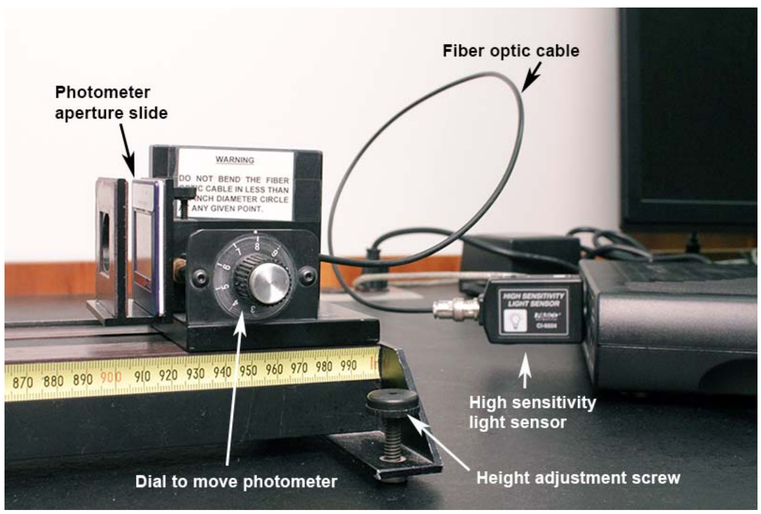
In this part of the experiment, we will locate only the minima of the interference pattern and measure the distance between them.

1. Open PASCO Capstone and choose the “Graph & Digits” option. Click the Hardware Setup tab to display the interface. Click on channel “A” and select “Light Sensor”. In the digits display box, click on “Select Measurement” and choose “Light Intensity”. Click “Record” to test out the sensor.

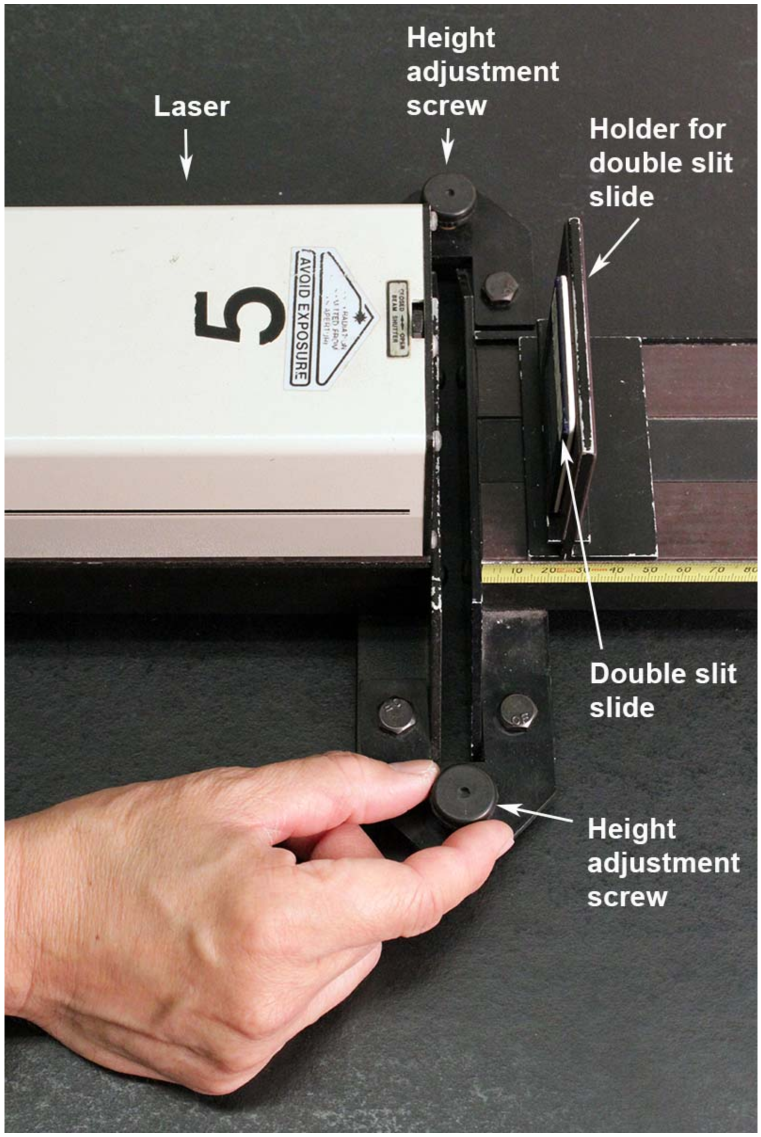
2. Look at the component carrier. A white line on the side of the carrier indicates the carrier position with reference to the meter scale on the optics bench. The white line is in the middle of the two vertical surfaces.

3. Study the translator carriage for a moment. At the back, a pointer line rides over a scale graduated in millimeters. One turn of the dial moves the pointer 1 millimeter, so the dial is reading in tenths of a millimeter. You can probably estimate hundredths of a millimeter on the dial scale. To begin aligning the system, move the translator carriage so the pointer is around the midpoint (approximately 24 mm) of the scale. Note there are three surfaces to which you can attach slides (see figure and image below). When using the Photometer Apertures slide, put it on the Rear Surface only, closest to the fiber optic cable. Also note there isn’t a white line to indicate the linear translator position with reference to the meter scale on the optics bench. Using the “front edge” of the translator as the indicator, the middle of the Rear Surface is offset by 4.8 mm.



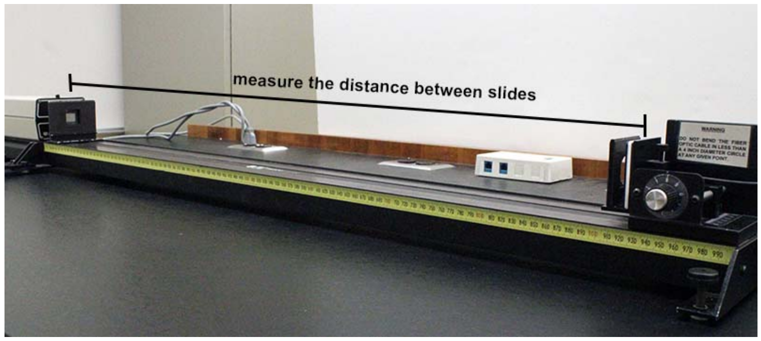


4. Start aligning the laser. Remove all slides (which attach magnetically to the carriers and the rear surface of the translator carriage) from the optics-bench setup. Turn on the laser and align the beam so it hits the center of the fiber-optic cable end. You can do this by adjusting the laser alignment bench screw at the back of the laser to set the beam at the right level so that when the translator carriage is moved by turning its dial, the end of the fiberoptic cable moves across the center of the laser beam.

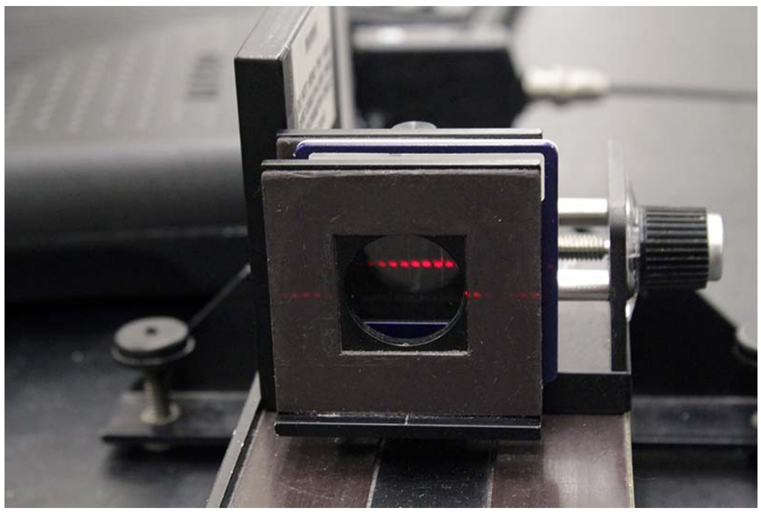


Be careful not to bump the laser, laser alignment bench, or optics bench when you are working later on the experiment so as to disturb the alignment. This is an easy mistake to make, especially with more than one person working on the experiment!

5. Place the Photometer Apertures slide on the rear surface of the translator stage. This slide has four single slits, but do not confuse it with the slides with single and double slits whose patterns you will be measuring. Position the 0.2-mm slit in front of the fiber optic terminal so that the laser beam is centered on it, shining into the fiber-optic cable end. The smaller the aperture you use the more detail you can detect in the pattern; however, you are also allowing less light into the light sensor and may not be able to detect the dimmest parts of the pattern. We suggest you use the 0.2-mm aperture for the double- and single-slit measurements below, but feel free to try other apertures if you think they would improve the results.



6. Position the narrowest double slit on the component carrier close to the laser so that when the laser is turned on, the double-slit fringe pattern is thrown onto the aperture slide. Note the modulated form of the pattern. Either start recording data or use "Monitor Data". Change the "Gain" on the light sensor to get a value between 1 and 5 volts.



7. Carefully record in the "Data" section the positions of the slit slide and the aperture slide using the optical-bench scale. The difference between these two positions is the distance L in Figure 2 and Eq. 3.

8. As you turn the dial of the translator stage, the aperture will move across the pattern. Measure and record the distances between the minima for five successive fringes carefully. Average the five distances. The translator stage may have "backlash": when reversing its direction, you need to turn the dial a perceptible distance before the stage begins to move. Therefore, turn the dial in only one direction when making the actual measurements.

9. Using Eq. 4, calculate the wavelength of the laser light. Compare it with the actual wavelength of 632.8 nm, and calculate the percentage error. (This is an atomic transition in neon atoms.)

PROCEDURE PART 2: POLARIZATION

1. The laser is not useful for polarization experiments because the laser beam is already partially polarized and the plane of polarization is rotating with time. You can check this by placing a polarizer between the laser and the fiber optic probe, and observing the Capstone reading.

Instead, set the “Gain” on the light sensor to “1”. Remove all slides, including the apertures slide. Put the Incandescent Light Source on the optics bench so the end with the light coming out is about 50 cm from the linear translator. Turn on the light source and adjust the bulb to get a bright beam to fall on the fiber-optic probe end. Place a polarizer on one of the component carriers and one on the front surface of the translator carriage.

2. Note that the polarizers are graduated in degrees and you can read off the angle from the marker on the bottom of the component carrier. Set both angles to zero for full transmission. Click “Record” to monitor the light sensor output. Adjust the gain of the light sensor if necessary.

3. Take intensity readings for every 10° of rotation of one of the polarizers from 0° to 90°.

4. Enter the angle and intensity data in two columns in Excel. In a third column, calculate $I_0 \cos\theta$. In a fourth column, calculate the theoretical intensity $I_0 \cos^2\theta$. Chart with Excel, and compare the experimental and theoretical curves. Is the cosine-squared curve clearly a better fit than the cosine curve? You may print out this Excel page for your records.

5. As a final polarization measurement, experiment with three polarizers. Record the data requested below in the “Data” section.

   a. Record the intensity of the light with no polarizers. You may have to change the gain on the sensor.

   b. Add one polarizer between the source and sensor, and record the intensity.

   c. Add a second polarizer, adjust for minimum intensity (crossed polarizers) and record the intensity.

   d. Now insert a third polarizer between the first two, and rotate it. For what angle of the middle polarizer (with respect to the first) does a maximum of light pass through all three polarizers?

   e. Record the intensity of light that passes through at the maximum position.

   f. Convert the measurement in step e to a decimal fraction of the total intensity (found in step a).

   g. What should the theoretical fraction be?

ADDITIONAL CREDIT PART 1: SINGLE SLIT (2 mills)

Measure the intensities of the side fringes, and compare them with the theoretical values.

1. Following the reasoning leading up to Eq. 8, calculate the intensity of the second side fringe as a decimal fraction of the intensity of the central maximum.

2. Set the “Gain” on the light sensor to “1”, and remove the incandescent light source. Recheck the laser alignment as in step (4) of the double-slit procedure. Position the apertures slide on the 0.2-mm slit.

3. Insert a single slit on one of the component carriers. Check that you are obtaining a nice single-slit pattern as in Figure 4 or 5 across the apertures slide.

4. In the "Data" section table, record the intensity of the central maximum, as well as the intensity of the first and second side fringes, in one column. You can locate the maxima by rotating the translator-stage dial while observing the light sensor output. In the second column, convert the intensities to a decimal fraction of the intensity of the central maximum. Record the theoretical value next to the results of the fractional intensities of the side fringes.

ADDITIONAL CREDIT PART 2 (3 mills)

Measure a single-slit intensity curve, and compare it with the theoretical curve.

1. Call up Excel on your computer. Enter the column of distance measurements for every 0.2 mm by using a "Series" operation.

2. Set up the narrowest slit on the optical bench. Start at the center of the central maximum, and record intensity readings every 0.2 mm past the second minimum (so that you cover the first side fringe) in the next column of Excel.

3. You need the slit width to calculate the theoretical curve from Eqs. 4 and 5. Measure this width with the traveling microscope.

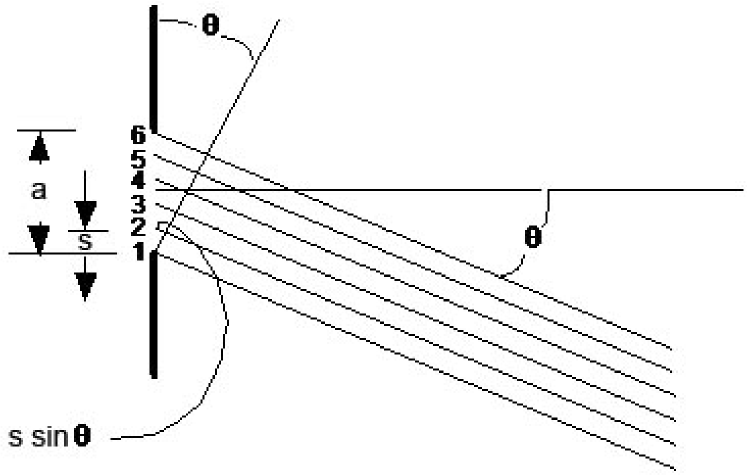
4. In the third column, compute the theoretical intensity from Eq. 4. Enter the formula correctly into the first cell; then use the "Fill Down" operation.

5. Graph your theoretical and experimental curves (normalized to the intensity at the center of the central maximum). If all looks well, you may print your chart out with the data to keep for your records.

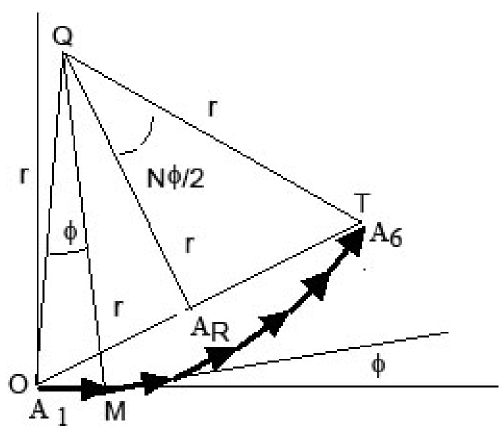
APPENDIX: SINGLE-SLIT THEORY CONTINUED

We will use a geometrical, or *phasor*, method to derive Eq. 5 for the intensity curve of the single-slit diffraction pattern.

Suppose that instead of a single beam of light passing through a slit of width a, there are N tiny sources of light (all monochromatic and coherent with each other) which are separated from each other by a distance s in such a way that $Ns = a$. We will start with six light sources so that our pictures are clear, but will eventually let N go to infinity.



In Figure 10, we are looking at the light rays coming from the sources that make an angle θ with the horizontal. Observing the figure closely, we see that the ray from source 2 travels a distance $s \sin\theta$ greater than the ray from source 1 en route to the viewing screen on the right (not shown in the figure). Thus, the light from a given source is out of phase with the light from the source just below it by a phase $(2\pi s \sin\theta)/\lambda$, which we denote ϕ for brevity. We will assign an amplitude A to each source and, with the phase difference ϕ, draw a diagram showing the addition of the light rays at some angle θ (Figure 11).



We see that each vector makes an angle ϕ (phase difference) with the preceding one, and the resultant vector is the total *amplitude* of light seen at angle θ. When we determine what OT $= A_{\mathrm{R}}$ is in terms of ϕ and A, we square the result to obtain the total *intensity* of light at angle θ.

Note that the vectors A_1 through A_6, each of equal magnitude A, lie on a circle whose radius is OQ $= r$. Since the angle OQM is ϕ, it follows that $A = |A_1| = 2r\sin(\phi/2)$ (some steps have been skipped here). But angle OQT is $N\phi$ (where $N = 6$ in this case), so $A_{\mathrm{R}} = 2r\sin(N\phi/2)$. Solving for A_{R} in terms of A and ϕ, we find

$$A_{\mathrm{R}} = A\sin(N\phi/2)\,/sin(\phi/2). \tag{10}$$

This is the amplitude of light from N slits, where $N = 6$ for the case we are illustrating. We now wish to let N go to infinity. As N approaches infinity, s approaches 0, but Ns approaches a (the slit width). Thus, $N\phi$ approaches Φ, the total phase difference across the entire slit:

$$\Phi = (2\pi a/\lambda)\,\sin\theta. \tag{11}$$

Thus, Eq. 10 becomes

$$A_{\mathrm{R}} = A\sin(\Phi/2)/\sin(\Phi/2N). \tag{12}$$

The angle $\phi = \Phi/N$ becomes infinitesimally small, so we can replace the sine term in the denominator of Eq. 12 with the angle itself:

$$A_{\mathrm{R}} = A\sin(\Phi/2)/(\Phi/2N). \tag{13}$$

Finally, $NA = A_{\mathrm{T}}$ (the total amplitude of light from the slit), so

$$A_{\mathrm{R}} = A_{\mathrm{T}}\sin(\Phi/2)/(\Phi/2). \tag{14}$$

If we let $\alpha = \Phi/2 = (\pi a/\lambda)\sin\theta$, then

$$A_{\mathrm{R}} = A_{\mathrm{T}}(\sin\alpha/\alpha). \tag{15}$$

The intensity is proportional to the square of the amplitude, so

$$I = I_0[(\sin\alpha)/\alpha]^2. \tag{16}$$

This is Eq. 5.

DATA

Procedure Part 1:

5. Slit-slide position = ______________________________

   Aperture-slide position = ______________________________

   L = ______________________________

6. Positions of 5 minima = ______________________________

   ____

   ____

   ____

   ____

   Average difference = ______________________________

7. Measured distance between inside edges = ______________________________

   Measured slit width = ______________________________

   d from measurements above = ______________________________

   Nominal d on slide = ______________________________

8. Calculated wavelength = ______________________________

   Percentage error = ______________________________

   Show your calculation of the wavelength neatly below.

Procedure Part 2:

3. You may pring out your data in Excel.
4. You may pring out your data in Excel.
5. a. Intensity with no polarizers = ______________________________

   b. Intensity with one polarizer = ______________________________

   c. Intensity with crossed polarizers = ______________________________

   d. Angle of middle polarizer = ______________________________

   e. Intensity with third polarizer = ______________________________

   f. Fraction = ______________________________

   g. Theoretical fraction = ______________________________

Additional Credit Part 1 Data:

	A	B	C	D
1	fringe	measured intensity	fractional intensity	theoretical intensity
2	central			
3	1st side			
4	2nd side			