Standing Waves

APPARATUS

- Computer and interface
- Mechanical vibrator
- Clip from vibrator to string
- Three strings of different densities
- Meter stick clamped vertically to measure vibration amplitudes
- Weight set
- Acculab digital scale

INTRODUCTION

Have you ever wondered why pressing different positions on your guitar string produces different pitches or sounds? Or why the same sound is produced by pressing certain positions on two or more strings? By exploring several basic properties of standing waves, you will be able to answer some of these questions. In this experiment, you will study standing waves on a string and discover how different modes of vibration depend on the frequency, as well as how the wave speed depends on the tension in the string.

THEORY: FORMATION OF STANDING WAVES

Consider a string under a tension F with its ends separated by a distance L. Figure 1 depicts a complex wave on the string, which could be produced by plucking the string or drawing a bow across it.

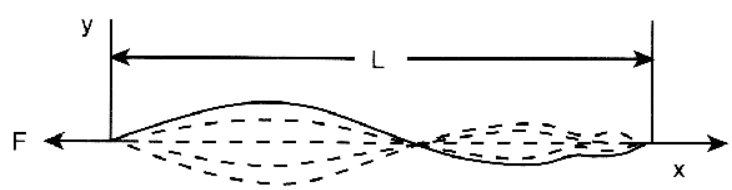


Figure 1

We will see that a complex wave such as this can be constructed from a sum of sinusoidal waves. Therefore, this focus of this experiment is on sinusoidal waves.

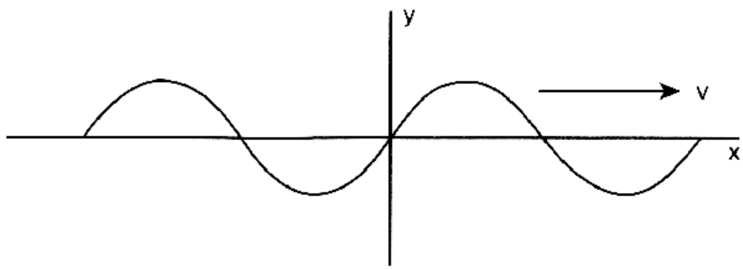


Figure 2

Figure 2 shows a wave traveling along the x-axis. The equation describing the motion of this wave is based on two observations. First, the shape of the wave does not change with time (t). Second, the position of the wave is determined by its speed in the x direction. Based on these observations, we see that the vertical displacement of the wave (y) is a function of both x and t. Let $y(x, t = 0) = f(x)$, where $f(x)$ represents the function that characterizes the shape of the wave. Then $y(x, t) = f(x - vt)$, where v is the speed of the wave. Although this description holds true for all traveling waves, we will limit our discussion to sinusoidal waves.

The vertical displacement of the traveling sinusoidal wave shown in Figure 2 can be expressed as

$$y(x, t) = A \sin[(2\pi/\lambda)(x - vt)], \tag{1}$$

where A is the *amplitude* of the wave (i.e., its maximum displacement from equilibrium) and λ is the *wavelength* (i.e., the distance between two points on the wave which behave identically). Expanding the term inside the brackets gives

$$y(x, t) = A \sin(2\pi x/\lambda - 2\pi vt/\lambda). \tag{2}$$

By substituting $\lambda = vT$, $k = 2\pi/\lambda$, and $\omega = 2\pi/T$ (where T is the *period*, k is the *angular wavenumber*, and ω is the *angular frequency*), we obtain

$$y(x, t) = A \sin(kx - \omega t). \tag{3}$$

As t increases, the argument of the sine function $(kx - \omega t)$ decreases. In order to obtain the same value of y at a later time, x must also increase, which implies that this wave travels to the right. Conversely, the argument $(kx + \omega t)$ represents a wave traveling to the left. When the right-traveling wave of Figure 2 reaches a fixed end of the string, it will be reflected in the opposite direction.

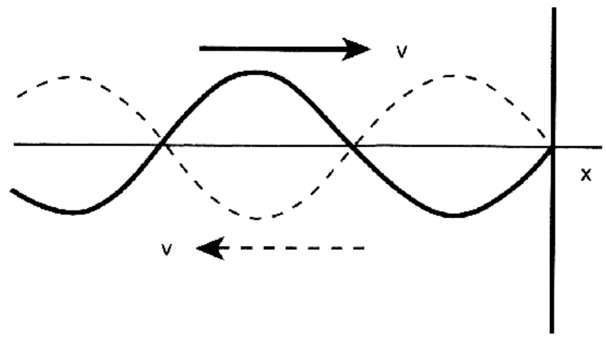


Figure 3

The right-moving incident wave, y_1, generates a left-moving reflected wave, y_2, with the same amplitude:

$$y_1(x,t) = A\sin(kx - \omega t) \tag{4}$$

$$y_2(x,t) = A\sin(kx + \omega t). \tag{5}$$

The resultant wave, y_3, which is the sum of the individual waves, is given by

$$y_3(x,t) = y_1(x,t) + y_2(x,t) = A\sin(kx - \omega t) + A\sin(kx + \omega t). \tag{6}$$

We can rewrite Eq. 6 by using the trigonometric identity:

$$A\sin(\alpha) + A\sin(\beta) = 2A\sin[(\alpha+\beta)/2]\ \cos[(\alpha-\beta)/2] \tag{7}$$

$$y_3(x,t) = 2A\sin(kx)\ \cos(\omega t). \tag{8}$$

Note that the x and t terms are separated such that the resultant wave is no longer traveling. Eq. 8 shows that all particles of the wave undergo simple harmonic motion in the y direction with angular frequency ω, although the maximum amplitude for a given value of x is bounded by $\mid 2A\sin(kx) \mid$. If we fix the two ends of the string and adjust the frequency so that an integral number of half waves fit into its length, then this *standing wave* is said to be in *resonance*.

The fixed ends impose a *boundary condition* on the string; its amplitude at the ends must be zero at all times. Thus, we can say that at $x = 0$ and $x = L$ (where L is the length of the string),

$$y_3(x=0,t) = y_3(x=L,t) = 0 \tag{9}$$

$$2A\sin(k\cdot 0)\ \cos(\omega t) = 2A\sin(kL)\ \cos(\omega t) = 0 \tag{10}$$

or

$$\sin(kL) = 0. \tag{11}$$

Eq. 11 is a boundary condition which restricts the string to certain modes of vibration. This equation is satisfied only when $kL = n\pi$, where n is the *index* of vibration and is equal to any positive integer. In other words, the possible values of k and λ for any given L are

$$kL = (2\pi/\lambda)L = n\pi \qquad (n = 1, 2, 3, \ldots) \tag{12}$$

or

$$\lambda = 2L/n. \tag{13}$$

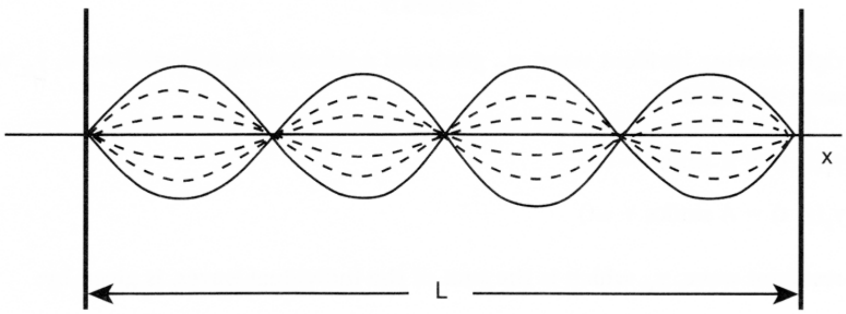


Figure 4

Figure 4 shows the mode with index $n = 4$: the fourth *harmonic*. The positions at which the vibration is small or zero are called *nodes*, while the positions where the vibration is largest are called *antinodes*. The number of antinodes is equal to the index of vibration and to the ordinal rank of the harmonic (fourth, in the case above). Figure 5 shows several other modes of vibration.

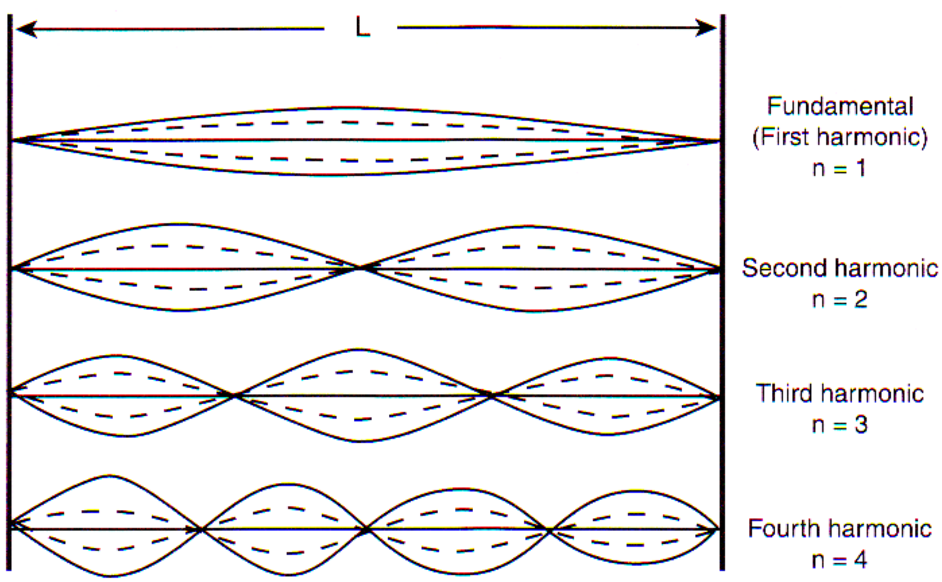


Figure 5

Note that wavelength = $2\times$(string length between supports)/n, where n = mode number (or index) = number of antinodes.

THEORY: PROPERTIES OF STANDING WAVES

The wave speed (v) depends on two quantities — frequency (f) and wavelength (λ) — which are related by

$$v = f\lambda. \tag{14}$$

A wave is created by exciting a stretched string; therefore, the speed of the wave also depends on the tension (F) and the mass per unit length of the string (μ). Physics texts give us the derivation of the wave speed:

$$v = (F/\mu)^{1/2}. \tag{15}$$

By combining Eqs. 13, 14, and 15, we can express the frequency as

$$f = v/\lambda = (n/2L)v = (n/2L)(F/\mu)^{1/2}. \tag{16}$$

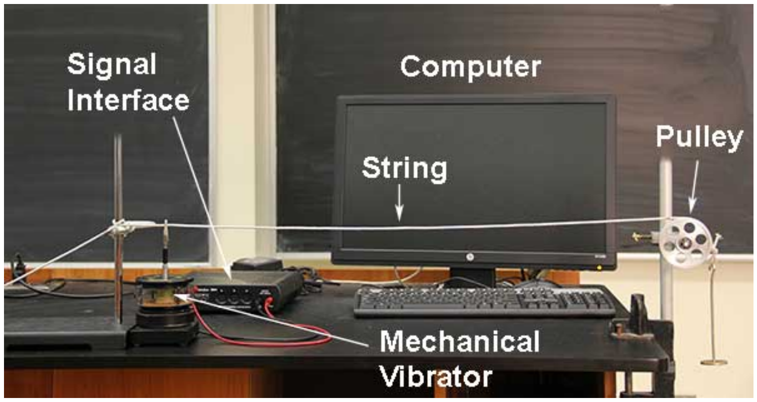
When a wave travels from one medium to another, some of its properties change (e.g., speed and wavelength), but its frequency remains fixed. For example, consider the point where two strings of different densities are joined. If the two strings have different frequencies, then the two sides of the point would oscillate at their own frequencies, and the point would no longer be a "joint". Mathematically speaking, the function describing the string would not be continuous at the "joint". The constancy of the frequency allows us to determine the wave speed and wavelength in a different medium, if the frequency in that medium is known.

EXPERIMENTAL SETUP

Set up the equipment as shown in Figure 6. Adjust the vibrator clamp on the side to position it firmly in the vertical orientation. Run one end of the string from a vertical bar past the vibrator and over the pulley. The vibrator is connected to the string with an alligator clip. Attach a mass hanger to the other end of the string. Throughout this experiment, you will be changing both the density of the medium (by using different strings) and the tension (by using different weights). To measure vibration amplitudes, it is helpful to have a meter stick clamped vertically near the string.

1. Turn on the signal interface and the computer.

2. Call up Capstone. Under "Hardware Setup", click on the output channels of the interface to connect the mechanical vibrator. Under "Signal Generator", click on "SW750 Output".



3. Note that a sine wave has already been selected. We will use only sine waves in this experiment. Set the amplitude and frequency of the signal generator initially to approximately +2 V and 20 Hz, respectively. Then click "On". You should see the string vibrate. Adjust the frequency to observe the multiple harmonics of the standing wave. Remember that you can obtain frequency steps of various sizes by clicking on the up and down arrows. Click the right and left arrows to adjust the size of these steps.

PROCEDURE: PART 1

In this section, we will keep the tension and density of the string constant to find experimentally the relationship between frequency and number of antinodes.

1. Adjust the frequency until you obtain a nice standing wave with two antinodes ($n = 2$). Record this frequency in the "Data" section.

2. Obtain and record the frequencies for consecutive n values. Take at least six measurements, starting with the fundamental mode.

3. Calculate and record the wavelength, Eq. 13, and wave speed, Eq. 14, corresponding to each n.

4. Plot a graph of frequency as a function of n. What is the relationship between the two variables?

PROCEDURE: PART 2

In this section, we will keep n constant and change the weights to find the relationship between frequency and tension.

1. Choose one of the three strings. If you like to see data that agrees well with theory, choose the finest string. If you would rather see more interesting data, for which you might need to explain the discrepancy, choose the most massive string. Measure and record the linear mass density ($\mu = M/L$) of the string by obtaining its total mass M and total length L. Use the digital scale to weigh the string. Keep all units in the SI system (kilograms and meters).

2. Using the 50-g mass hanger, measure and record the frequency for the $n = 2$ mode. (Note: You may choose any integer for n, but remember to keep n constant throughout the rest of this section.)

3. Add masses in increments of 50 g, and adjust the frequency so that the same number of nodes is obtained. Take and record measurements for at least six different tensions.

4. The wave speed should be related to the tension F and linear mass density μ by $v = (F/\mu)^{1/2}$. Calculate and record the wave speed in each case using Eq. 14, and plot v^2 as a function of F/μ. (You have calculated v^2 from the frequency and wavelength; these are the y-axis values. You have calculated F/μ from the measured tension and linear mass density; these are the x-axis values. Be sure to convert the tension into units of Newtons.) You now have the experimental points.

5. Now plot the "theoretical" line $v^2 = F/\mu$. This is a straight line at 450 on your graph, if you used the same scale on both axes. Do your experimental and theoretical results agree well? If not, what might be the reasons?

PROCEDURE: PART 3

In this section, we will determine the relationship between frequency and the density of a medium through which a wave propagates.

1. Measure the linear mass densities ($\mu = M/L$) of the two other strings as described above.

2. Keeping the tension and mode number constant at, say, 100 g and $n = 2$, measure and record the frequencies for the three strings.

3. Calculate and record the "experimental" wave speed from the frequency and wavelength for each string density.

4. Calculate and record the "theoretical" wave speed for each string density from $v = (F/\mu)^{1/2}$, and compare these speeds with the experimental values.

DATA

Procedure Part 1:

1. Frequency ($n = 2$ mode) = ______________________________

2. Frequency ($n = 1$ mode) = ______________________________

   Frequency ($n = 2$ mode) = ______________________________

   Frequency ($n = 3$ mode) = ______________________________

   Frequency ($n = 4$ mode) = ______________________________

Frequency ($n = 5$ mode) = ______________________________

Frequency ($n = 6$ mode) = ______________________________

3. Wavelength ($n = 1$ mode) = ______________________________

   Wave speed ($n = 1$ mode) = ______________________________

   Wavelength ($n = 2$ mode) = ______________________________

   Wave speed ($n = 2$ mode) = ______________________________

   Wavelength ($n = 3$ mode) = ______________________________

   Wave speed ($n = 3$ mode) = ______________________________

   Wavelength ($n = 4$ mode) = ______________________________

   Wave speed ($n = 4$ mode) = ______________________________

   Wavelength ($n = 5$ mode) = ______________________________

   Wave speed ($n = 5$ mode) = ______________________________

   Wavelength ($n = 6$ mode) = ______________________________

   Wave speed ($n = 6$ mode) = ______________________________

4. Plot the graph of the frequency as a function of n using one sheet of graph paper at the end of this workbook. Remember to label the axes and title the graph.

Procedure Part 2:

1. Mass of string 1 = ______________________________

   Length of string 1 = ______________________________

2. Frequency (with 50-g mass) = ______________________________

3. Frequency (with 100-g mass) = ______________________________

   Frequency (with 150-g mass) = ______________________________

   Frequency (with 200-g mass) = ______________________________

   Frequency (with 250-g mass) = ______________________________

   Frequency (with 300-g mass) = ______________________________

   Frequency (with 350-g mass) = ______________________________

4. Wave speed (with 50-g mass) = ______________________________

   Wave speed (with 100-g mass) = ______________________________

   Wave speed (with 150-g mass) = ______________________________

   Wave speed (with 200-g mass) = ______________________________

   Wave speed (with 250-g mass) = ______________________________

   Wave speed (with 300-g mass) = ______________________________

   Wave speed (with 350-g mass) = ______________________________

   Plot the experiment graph of v^2 as a function of F/μ using one sheet of graph paper at the end of this workbook. Remember to label the axes and title the graph.

5. Plot the theoretical graph of v^2 as a function of F/μ using the same sheet of graph paper. Remember to label the axes and title the graph.

Procedure Part 3:

1. Mass of string 2 = ______________________________

   Length of string 2 = ______________________________

   Mass of string 3 = ______________________________

   Length of string 3 = ______________________________

2. Frequency (string 1) = ______________________________

   Frequency (string 2) = ______________________________

   Frequency (string 3) = ______________________________

3. Experimental wave speed (string 1) = ______________________________

   Experimental wave speed (string 2) = ______________________________

   Experimental wave speed (string 3) = ______________________________

4. Theoretical wave speed (string 1) = ______________________________

   Theoretical wave speed (string 2) = ______________________________

   Theoretical wave speed (string 3) = ______________________________

   Percentage difference between experimental and theoretical speeds (string 1) = __________

Percentage difference between experimental and theoretical speeds (string 2) = __________

Percentage difference between experimental and theoretical speeds (string 3) = __________

ADDITIONAL CREDIT PART 1 (1 mill)

Carefully write out a complete answer to the question posed at the beginning of the experiment: Why does pressing different positions on your guitar string produce different pitches?

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ADDITIONAL CREDIT PART 2 (2 mills)

As you tune the frequency, there is a resonance of sorts at each higher mode of vibration. That is, as you tune the frequency, the amplitude of vibration is very large when you are at the correct frequency for the mode, but becomes smaller as you move away from the correct frequency, until you begin to approach the frequency of the next mode. The response might look similar to the graph below.

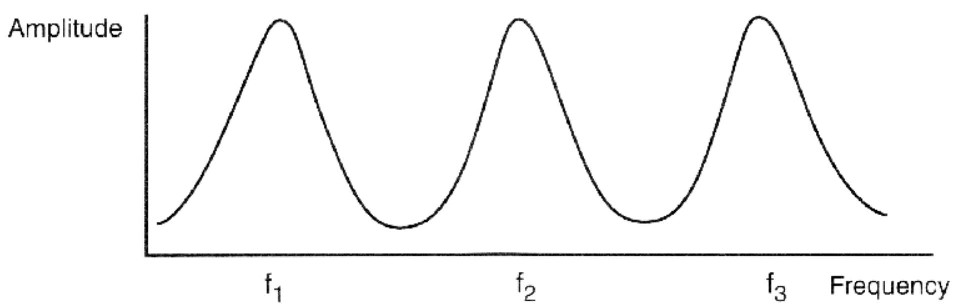

Figure 7

We want to measure the quality factor, Q, of one of these resonances and study how the oscillations decay. Refer to the discussion of Q in Experiment 1.

Choose a mode of vibration where you can get a nice large amplitude (e.g., $n = 2$, with 200 g on the heavy string). Clamp a vertical meter stick near one of the antinodes so you can measure the amplitude of vibration. Measure carefully near the resonance maximum, and record the frequencies on either side of resonance when the amplitude has fallen to $1/\sqrt{2}$ of its maximum value. Using

$$Q = \omega_1/\Delta\omega = f_1/\Delta f, \qquad \text{(Eq. 16 in Experiment 1)}$$

where $f = \omega/2\pi$, determine Q from your measurements.

As discussed in Experiment 1 in connection with its Eq. (16), Q also controls the damping rate of the vibration. Start the wave motion until it builds up to full amplitude. Then switch off the driving vibrator, and observe the wave motion decay. Measure the full amplitude A_{max} of vibration at resonance by reading off distances from the meter stick while the vibrator is driving the wave, and calculate A_{max}/e ($e = 2.718\ldots$). Devise a way to note this reduced amplitude on the meter stick.

Start the wave again at full amplitude, switch off the drive, and measure the time required for the amplitude to decay to A_{max}/e (the so-called "e-folding time"). Compare this time with $2Q/\omega_1 = Q/\pi f_1$, and record the results below.

Amplitude at resonance (A_{max}) = ______________________________

$A_{\text{max}}\sqrt{2}$ = ______________________________

Frequencies at which amplitude is equal to $A_{\text{max}}\sqrt{2}$ = ______________________________

Difference in frequencies (Δf) = ______________________________

Q = ______________________________

A_{max}/e = ______________________________

Time required for amplitude to decay to A_{max}/e = ______________________________

$2Q/\omega_1$ = ______________________________